Abstract. The paper aims at analyzing the dynamics of the housing prices treated as a time-series of a stochastic process. Obtained results show that in a longer perspective these prices form a nonstationarity process characterized by time-variant statistical descriptors. On the other hand, the prices analyzed within much shorter time windows appear to remain stationary, and are found to be developed in a random walk process. In such a case, the price distribution is close to normal distribution.

Keywords: real estate market, dynamics, nonstationarity

JEL Classification: C51, R30, R32

INTRODUCTION

Real estate markets are dynamic systems with high inertia relative to other economic systems, which means that they react much slower to changes in external conditions. The inertia of real estate markets is best evidenced during periods of rapid changes, when they undergo deep structural transformations within a short time, whereas the pace of changes during long-term evolution is slow enough to ensure a state of system equilibrium. According to Brzezicka and Wiśniewski (2013), it can be considered that the real estate market is a special type of market with its own rules, and far from the definition given by mainstream economics.

The paper, as a part of wider research project, deals with the real estate market seen as a dynamic and open system in perpetual pursuit of equilibrium in order to explain observed changes in housing prices. If so, the market can exchange signals with other systems (open system), and its evolution can be described using mathematical equations (dynamic system). To overcome this problem we have previously used the catastrophe theory and critically damped harmonic oscillator. From an intuitive point of view, global evolution of the system is manifested in a series of continuous changes intertwined with sudden changes of different character. Results of our previous works (Belej and Kulesza 2013) confirm the assumption that sharp changes in property prices significantly affect the structure of the entire real estate market, and that quasi-discrete
changes should be regarded as critical points in the market evolution. On other hand, we also demonstrated (Belej and Kulesza 2014) that local real estate markets are coupled together forming larger system, in which interactions propagate globally. It means that they form a system of communicating vessels. It is possible, therefore, that under very specific circumstances instability can spread from one local real estate market to the others generating ‘the waves of instability’, but on the other hand, the markets can also react collectively in order to suppress the instability.

The main goal of the research is an analysis of time series of real estate prices on local markets under assumption that there is a pattern in the time series, which is a fingerprint of the market dynamics. The main reason to undertake proposed study originated from the observation of steep price changes in Poland during most recent quarter century, i.e. large increase in dwellings prices from 2006 until 2007 preceded by otherwise stable evolution. According to the authors, dynamic process observed on the Polish real estate market have led to instability. The market adjusted its trajectory of evolution by moving from a near-balanced state to states far from equilibrium in a process of discontinuous changes in the system's state variable. In most studies (Brzezicka and Wiśniewski 2014, Drchal 2014, Dittmann 2013, Forýs 2011, Kokot and Bas 2013, Renigier-Biłożor et al. 2014), the interval from 2006 to 2007 is considered a period of rapid increase in property prices, when evolution turns from stable to unstable. According to the authors, however, symptoms of this transition appeared much earlier.

In this research we analyzed the identification of the nonstationarity type in time series of real estate prices in Poznan in Poland. This market is both representative of other local real estate markets and is strongly associated with the national economics. While, a hypothesis to test, it is a possibility to identify of critical points of the evolution of local markets for identification distinguished phases of the evolution of local markets. This can be done using the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF).

**RESEARCH METHODOLOGY AND PROCEDURES**

There are numbers of reasons for studying time series. These include characterization of time series (or signals), understanding and modeling the data generating system, forecasting of future values and optimal control of a system [Madsen 2007]. In the area of economics, time series analyses are used usually for: economic forecasting, sales forecasting, stock market analysis, yield projections, inventory studies, budgetary studies and real estate market analysis. Issues related to the time series have been the subject of many studies, among others Chatfield [1975], Kendall [1976], Lange [1975], Zeliaś [1984], Miło [1990] and Trojanek [2008].

According to Madsen [2007], time series analysis deal with statistical methods for analyzing and modeling an ordered sequence of observations. This modeling results in a stochastic process model for the system which generated the data. The ordering of observations is most often, but not always, through time, particularly in terms of equally spaced time intervals.

The strength of relations and the inertia of stochastic processes can be characterized using joint distribution of random variables and statistical descriptors, for example: mean, variance, and autocovariance. Depending on the way the distribution parameters evolve, one can distinguish between various stochastic processes. One of the most basic methods of describing the series of data, is to check whether it is stationary (either strongly or weakly), or nonstationary.

A nonstationary stochastic process is a process in which the probability distribution function varies over time. The time series of stochastic process may be characterized by nonstationarity in average and nonsta-
tionarity in variance. Time series, which are nonstationary in their mean value, tend to vary unidirectionally towards steady increase or decrease. Time series nonstationaries in their variance are in turn highly fluctuating.

A stochastic process is stationary when it is not characterized by time correlations and when variables have identical probability distribution. Time series is said to be (weakly) stationary if [Hamilton 1989; Phillips 2001]:

Its expected value is constant over time and time-independent:

\[ \forall_t E(X_t) = \mu; \]

Its variance is constant over time, time-independent and finite:

\[ \forall_t E(X_t^2) = \sigma^2 < \infty; \]

Its autocovariance depends only on the distance between samples:

\[ \forall_{t, k, s} \text{cov}(X_t, X_{t+k}) = \text{cov}(X_{t+s}, X_{t+k+s}); \]

where:
- \( E(X) \) – expected value
- \( \sigma^2(X) \) – variance
- \( t \) – time

To check whether or not the process is stationary, one must evaluate parameters of its distribution. However, they are usually unknown, but it is still possible to verify if the parameters of multidimensional distribution of a random variable remains constant. In analysis of stationarity or nonstationarity of time series in housing prices we decided to use both methodologies.

In a first step, the verification of constant values of mean \( E(X_t) \) and variance \( \sigma^2(X_t) \) was carried out. In a second step, the analysis of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) were conducted. Autocorrelation Function (ACF) is equivalent to normalized autocovariance function:

\[ \text{corr}(X_t, X_{t+k}) = \frac{\text{cov}(X_t, X_{t+k})}{\sqrt{\text{cov}(X_t, X_t) \text{cov}(X_{t+k}, X_{t+k})}}; \]

and indicates the relationship between distant observations. Autocorrelation function graph for stationary process, with increasing delay, should rapidly decrease. Partial Autocorrelation Function (PACF) is the autocorrelation between \( X_t \) and \( X_{t+k} \) with the linear dependence of \( X_{t+1} \) through to \( X_{t+k-1} \) being removed. Partial Autocorrelation Function evaluates the delay amount of the analyzed process for autoregressive model (AR).

Scientific literature lacks of original works on discrimination between stationarity and nonstationarity on real estate market. The nonstationarity and autocorrelation in real estate time series has been found by Webb et. Al. (1992). Chaudhry et al. (1999) suggest that nonstationarity in the time series for office, retail, and industrial properties is significant when both drift and trend are included in the model. However, progress in cointegration methodologies provide an alternative framework for investigating equilibrium price adjustments in financial and real estate time series, especially their long-run relationships. According to Wisniewski (2009), analyzed time series of property prices are roughly nonstationary in average, and increment calculation results in overfitting. In this analysis, ADF, PP and KPSS tests has shown that the time
series of property prices are not integrated, implying that it may not be transformed into a stationary time series through increment calculation.

RESEARCH SUBJECT - DWELLING PRICES IN POZNAN (2001-2011)

The study was carried out in the city of Poznan in Poland. The Registry of Prices and Values maintained by the City Administration Office was the source of data relating to about 12,000 property transactions (dwellings) in Poznan. Time series of housing prices were analyzed for the period between January 2001 and December 2011. Fig. 1 shows the plot of calculated monthly-averaged (per square meter) mean of transaction prices and histogram of transaction prices computed for whole time interval.

According to Fig.1 (left side) property prices in Poznan from 2001 till 2004, increases slowly and stable, then we had a rapid increase of prices and after this the real estate market gradually stabilized during a gently downward trend in 2008-2011. In our opinion changes in monthly averaged mean of housing prices exhibit a housing turbulence (price bubble) emerging from otherwise slow, long-lasting trend. The distribution of the prices (Fig. 1 – right side) has a bimodal character. In strongly bimodal systems, the evolutionary path can take the shape of a hysteresis loop which moves between the allowed states of equilibrium; alternatively, it can be split into two paths that evolve more or less independently. In first step of procedure for analysis of stationarity and nonstationarity of real estate prices time series we conducted the verification of constancy of expected value - price mean (E(Xt)) and variance $\sigma^2(X_t)$.

Additionally we analyzed the volume of transactions calculated for each month within analyzed period. Fig. 2 shows monthly averaged (from 2001 until 2011) data for the variance, number of transactions, skewness, and kurtosis with respect to the arithmetic mean.
The variance of housing prices is obviously correlated with changes in the mean values in the sense that they sharply increased during the turbulence (bubble) and gradually decrease afterwards. In some approximation can it be concluded that the mean and the variance is constant in some periods, before and after the turbulence period.

The variance of housing prices is obviously correlated with changes in the mean values in the sense that they sharply increased during the turbulence (bubble) and gradually decrease afterwards. In some approximation can it be concluded that the mean and the variance is constant in some periods, before and after the turbulence period. For the whole time period from 2001 to 2011, the hypothesis of the constancy of the distribution parameters is insignificant. If so, the time series of housing prices, taken as a whole, forms a nonstationary stochastic process. The number of transactions turns out to be periodic, and correlated with changes in housing prices.

In second step of procedure for analysis of stationarity and nonstationarity of real estate prices time series we conducted the verification with the use of autocorrelation function (ACF) and partial autocorrelation function (PACF). Fig. 3 shows the plots of the autocorrelation function (ACF) and partial autocorrelation function (PACF).
If the process is stationary, the plot of the ACF should quickly decay with increasing lag. Correlogram of time series of housing prices from 2001 until 2011 remains non-zero even for largely delayed samples (with respect to the series length). The plot of ACF decreases below significance level no earlier than for \( t = 40 \).

Graphical analysis of the plot of the ACF allows us to determine the turning points in the market evolution. We consider that the time series of monthly-averaged means reveal the presence of two intertwined processes: (A) stable, long-term growth trend (two periods \( t \) from 1 to 40 and \( t \) from 86 to 132) and (B) sharp, short-term changes specific of the price turbulence (one period \( t \) from 41 to 85). Hence, the evolution of the housing prices can be schematically represented as a sequence A/B/A. Fig. 4, 5, 6 show results of calculations performed within the above defined time windows.

Conclusion for ACF/PACF for \( t \) from 1 to 40 (A): stationary process, prices thought of as a white noise process with constant mean, price distribution close to normal (lightly skewed towards higher values), mean price equivalent to the equilibrium level, negligible influence of past independent variables (both macro- and microeconomic), short-term system memory.

Conclusion for ACF/PACF for \( t \) from 41 to 85 (A): price thought of as a random walk process, increase in mean and variance values, prices in search of new equilibrium level, significant changes in the surrounding systems, history of independent variables becomes extremely important (both macro- and microeconomic), exploited history of surrounding systems.
Figure 4. Plot of autocorrelation function (ACF) and partial autocorrelation function (PACF) for the A period (t from 1 to 40)
Source: Own elaboration.

Figure 5. Plot of autocorrelation function (ACF) and partial autocorrelation function (PACF) for B period (t from 41 to 86)
Source: Own elaboration.
Descriptive analysis on nonstationarity of the time series on real estate market

Figure 6. Plot of autocorrelation function (ACF) and partial autocorrelation function (PACF) for the period (t from 86 to 132)

Source: Own elaboration.

Conclusion for ACF/PACF t from 85 to 132 (A): stationary process, prices thought of as a white noise process with constant mean, price distribution close to normal (lightly skewed towards higher values), mean price equivalent to the equilibrium level, negligible influence of past independent variables (both macro- and microeconomic), short-term system memory.

CONCLUSION

Real estate markets turn out to be highly inertial, and fully exhibit this property during sharp changes of their state. Characteristic relaxation time is longer than in other economical systems). Discrimination between stationary/nonstationarity parts of the time series of real estate prices was carried out using an autocorrelation function (ACF) and partial autocorrelation (PACF). With this procedure we found identification of critical points of the evolution of local markets possible.

Time series of the following statistical parameters: mean, geometric mean, median, variance, data range, kurtosis, and skewness, all reveal the presence of two intertwined processes: (A) stable, long-term growth trend and (B) sharp, short-term changes specific of the price turbulence (bubble). Taking into consideration time series of housing prices in the subintervals (A/B/A) one can see that: subinterval A: stationarity; short-term system memory, subinterval B: nonstationarity; significance of long-term history of independent variables.
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