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Forecasting 15-year development differences for bankrupt and non-bankrupt enterprises in Asia, Latin America and Europe

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Abstract. The study proposes dynamic models to address the increased importance of the corporate bankruptcy phenomenon and the growing need for reliable forecasting models for three economic regions: Central Europe, Far-East Asia and Latin America. The paper presents a solution for effective bankruptcy risk forecasting by implementing both highly effective and usable trajectories that take into account the dynamic long-term economic indicators of enterprises. The objective of the research is to develop separate models that forecast the long-term differences in the economic performance between bankrupt and non-bankrupt enterprises in Latin America, Far-East Asia and Europe. A financial engineering approach was implemented to achieve this goal. Nine forecasting models were developed with the use of three methods: a multivariate discriminant analysis, the logit model, and fuzzy sets. With the use of the estimated models, 15-year dynamic trajectories were determined for each group of enterprises. The estimated models and dynamic systems in the form of trajectories can be used by analysts to identify the symptoms of a company going bankrupt long before the traditional, static bankruptcy forecasting models reveal such risk, thus they can be implemented in decision-making processes in economic systems.

Keywords: development differences, financial crisis, forecasting, fuzzy sets, bankruptcy models, decision making

JEL Classification: G33, F37

1. INTRODUCTION

The global financial crisis of 2007-2012, the COVID-19 pandemic, and the ongoing war in Ukraine have dramatically increased the risk of corporate bankruptcies worldwide. All three crises have negatively affected the financial performance of enterprises through increased interest rates, rapid inflation, volatile exchange rates, and other significant macroeconomic factors.

Forecasting the bankruptcy of enterprises has been a subject of significant interest to researchers, financial analysts, managers, and bankers for decades. Since the development of the first multidimensional discriminant analysis model by Altman (1968), the number of studies on models predicting a company's collapse has increased dramatically, with many articles published globally on this subject. Many models have been proposed, starting with the development of statistical models (e.g. Barboza et al., 2017; Boda & Uradnicek, 2016; Hajek et al., 2014; Ho et al., 2013; Hosmer et al., 2013; Jackson & Wood, 2013; Lukason & Hoffman, 2014; Voda et al., 2021), through to the implementation of artificial intelligence methods very often originating from scientific disciplines as different as biology or mechanics (e.g. Acosta-Gonzales & Fernandez-Rodriguez, 2014; Hosaka, 2019; Jabeur et al., 2021; Jardin, 2015; Oenniche et al., 2018; Sun et al., 2014; Tsai, 2014).

Despite considerable progress in research regarding corporate bankruptcy forecasting models, few studies have focused on the problem of the forecasting horizon. Most researchers have explored the forecasting of the financial failure of enterprises and the prediction of the risk of insolvency one or two years prior to the declaration of bankruptcy. Such studies have mainly focused on minimising classification errors (Types I and II) and maximising the overall effectiveness of the model using certain financial variables.

To fill this gap in the literature, this study aims to develop dynamic systems that forecast long-term differences in economic performance between bankrupt and non-bankrupt enterprises in Latin America, Far-East Asia and Europe. The idea behind this objective is to investigate also the long-term differences in the development of "good" and future "bad" firms. Although forecasting models cannot predict horizons longer than two to three years before financial failure, an important and still unanswered question in the literature is whether the models can be used to develop dynamic systems in the form of trajectories. Such trajectories could extend the forecast period to up to 10-15 years before the failure. Only by an understanding of the long-term dynamic trajectory and by early identification of the premonitory signals of distress can managers create various scenarios to avoid a financial crisis.

The contribution of the paper is threefold. *First*, it develops three bankruptcy forecasting models for each specific economic region of enterprises. The author developed three populations of firms: one from Latin America (from such countries as Mexico, Argentina, Chile, Peru, Brazil and Colombia), one from Europe (from such countries as Germany, France, Spain, Finland, Italy, Poland and Sweden), and one from Far-East Asia (from such countries as Taiwan, Japan, China, South Korea and Malaysia). Such a research approach will allow not only the development of three specialised models for three different economic regions, but also the evaluation of which forecasting method is characterised by the smallest decrease in effectiveness in regard to the increasing horizon of the forecast. *Second,* it implements a dynamic approach to forecasting bankruptcy risk by developing 15-year trajectories of bankrupt and non-bankrupt firms. *Third,* it investigates long-term differences in the symptoms of going bankrupt that can be identified for companies in Latin America, Asia and Europe.

The paper consists of six sections. In the Introduction, the author justifies the topic, the study objectives and the contributions and innovations to the literature. Section 2 presents an overview of studies on the phases of the process of bankruptcy. Section 3 introduces this study's assumptions. In Section 4, the author presents nine bankruptcy prediction models. Section 5 discusses the developed dynamic systems for enterprises in three regions of the world and presents the results of effectiveness tests. Section 6 concludes the paper.

2. LITERATURE REVIEW

Business failure is considered the result of an evolutionary process (Flores-Jimeno & Jimeno-Garcia, 2017; Gonzalez-Bravo & Mecaj, 2011). In fact, the financial distress of a company is a dynamic ongoing process and is the result of the continuous abnormality of a business operation for a period of time (Sun et al., 2014). The literature review conducted by the author shows that the process and phases of business failure and the nature of the causes of the bankruptcy of enterprises is not dependent on the region of the world in which a firm operates (for example, Amankwah-Amoah, 2016; Hercacleous & Werres, 2016; Flores-Jimeno & Jimeno-Garcia, 2017; Lukason & Hoffman, 2014; Lukason & Laitinen, 2016; McGovern, 2007; Mellahi & Wilkinson, 2004). General conclusions can be drawn on the process and causes of corporate bankruptcy by combining the results of different studies.

In most cases, bankruptcy is a continuous process that can be divided into several stages: the appearance of the first signs of crisis, blindness and ignorance of the financial and nonfinancial symptoms of the economic crisis until the final phase of the crisis, insolvency. This phenomenon is neither sudden nor impossible to predict. Therefore, when the warning signs are recognised early, the company's management will have more time for decisions in the subsequent phases of the crisis to prepare for and reduce the impact of the financial crisis.

The literature distinguishes between three and five stages of financial crisis, ending in the bankruptcy of a company. The first author to distinguish such stages was Fitzpatrick, who in 1934 described the following stages of crisis regarding the studied failed companies (Fitzpatrick, 1934):

- Stage one: "crisis incubator". According to Fitzpatrick, this phase appears unnoticed by the company management. In this phase the first financial difficulties appear.
- Stage two: "financial embarrassment". At this stage the management recognises problems concerning the company's financial condition, but due to the limited capacity of any immediate response to cash needs, it does not take decisive corrective action. In addition, if assets are not liquid enough to meet current obligations, this phase can quickly move into the third phase of crisis.
- Stage three: "financial insolvency". This phase develops with an increase in current liabilities, a decrease in the liquidity of assets, and less ability to regulate the payment of trade and bank credits. According to Fitzpatrick, enterprises in this phase of crisis still have a chance to evade it and succeed. However, doing so requires the determined efforts of the management. In the absence of decisive action, the company enters stage four of the crisis.
- Stage four: “complete insolvency”. This phase occurs when liabilities exceed assets. The company will be unable to avoid bankruptcy.
- Stage five: "bankruptcy". The company publicly announces its bankruptcy.

On the other hand, Argenti (1976) distinguishes four stages of deterioration of a company's financial situation:

- Stage one: Business deficits gradually become consolidated, although they still do not cause significant disturbances.
- Stage two: Deficiencies lead to errors and increasing irregularities.
- Stage three: Problems from stage two reveal significant business disruption, especially in regard to the firm's solvency.
- Stage four: The firm goes bankrupt and must liquidate.

Like Argenti, Richardson, Nwankwo and Richardson (1994) observe four phases of financial crisis in enterprises:

- Stage one: "negation of crisis". The management is blind to the first warning signs.
- Stage two: "denial of crisis". Even though symptoms of the crisis are already perceived by the management, the situation is believed to be temporary without the requirement for remedial actions.
- Stage three: "disorganisation". Corrective steps are taken in the enterprise, but they are insufficient, and the size and consequences of the crisis are underestimated.
- Stage four: "collapse of the organisation". The firm no longer has the adequate capacity to overcome the difficulties and management structures break down, resulting in the company's bankruptcy.

Similarly to the previously mentioned authors, Sharma and Mahajan (1980) and Kash and Darling (1998) also recognise four phases of crisis leading to corporate bankruptcy:

- In the first phase, initial errors of inefficient and poor management are made.
- In the second phase, further mistakes can be noticed in strategic planning and errors regarding the inadequate implementation of strategic plans. According to Kash and Darling, the second phase provides the greatest opportunity to counter the financial crisis in the company.
- In the third phase, due to mistakes made in the previous two phases and insufficient measures taken in the second phase of crisis, a general decline in all financial ratios is noticeable.
- In the fourth phase, the enterprise goes bankrupt.

Although the phases of the gradual financial failure of enterprises do not differ between firms from different regions, the process of decline can vary in length and time (Lukason & Hoffman, 2014). Because the literature lacks studies on this topic, the diversity of trajectories leading to bankruptcy must be determined (according to the region of the world). In some regions, the trajectory can be more chaotic or more gradual than in others. The difference is due to the different impacts of negative factors coming from three sources—the macroenvironment, the meso-environment and the company—on the financial situation of an enterprise in various stages of financial crisis. In each crisis phase, these factors can differently affect a company or cause chain reactions as the collapse of a company is affected by many overlapping factors (Figure 1).

As can be seen in Figure 1, the risk of bankruptcy depends on three factors: (1) macroeconomic factors, such as the overall economic and political situation of the country or the fiscal, monetary and exchange rate policy of the government; (2) meso-environmental factors, such as the lack of customers and/or suppliers, increased competitive pressure, weak cooperation with banks, a change in manufacturing technology, or the lack of market protection; and (3) microeconomic factors, such as an inefficient decision-making process, the inability to adapt and manage risk, the lack of knowledge and experience in management, marketing and finance, insufficient knowledge about the industry in which the firm operates, poor capacity to manage or resolve conflicts within a team or to communicate with employees, or the extensive use of financial leverage.

Summing up, enterprise bankruptcy is a dynamic process which can be impacted by different endogenous and exogenous factors. The nature and force of these factors depend on the region of the world, but research is lacking that would allow us to identify and evaluate long-term differences in the process of going bankrupt separately for firms in different world economic regions. Moreover, with increased globalisation, faster economic changes, and the new dimension of increased financial risk in the context of the last three worldwide crises, we should now focus on prolonging the forecasting horizon to 10 or even 15 years prior to the announcement of bankruptcy using the knowledge on the above-mentioned phases of going bankrupt. As Jardin (2017) states, the accuracy of failure models at a horizon exceeding one year can be considered a real challenge for financial institutions. At a horizon of more than one year, the models' accuracy decreases substantially (Jardin & Severin, 2011). For example, Altman's model accuracy rate decreases from 95% one year before failure to 48% three years before failure (Altman, 1968), and Sharma and Mahajan's model decreases from 91.7% to 73.9% in the same period (Sharma and Mahajan, 1980). According to Jardin and Severin (2011), regardless of the modelling technique (linear or nonlinear, regression or classification), models always have the same drawback of a short forecasting horizon. Another shortcoming of traditional forecasting models is their stationarity (Korol, 2020). Bankruptcy models are usually estimated with the use of financial ratios calculated with data from balance sheets and income statements (Korol, 2018). Such models do not take into account changes in the environment nor any dynamic approach to financial ratios.

Moreover, predicting the bankruptcy of companies is imprecise and ambiguous. The process of business failure is affected by many internal and external factors that cannot be precisely and unambiguously defined. That is why the available forecasting models in the literature are characterised by such short-term predicting properties. By implementing system dynamics, we can better handle dynamically complex problems.

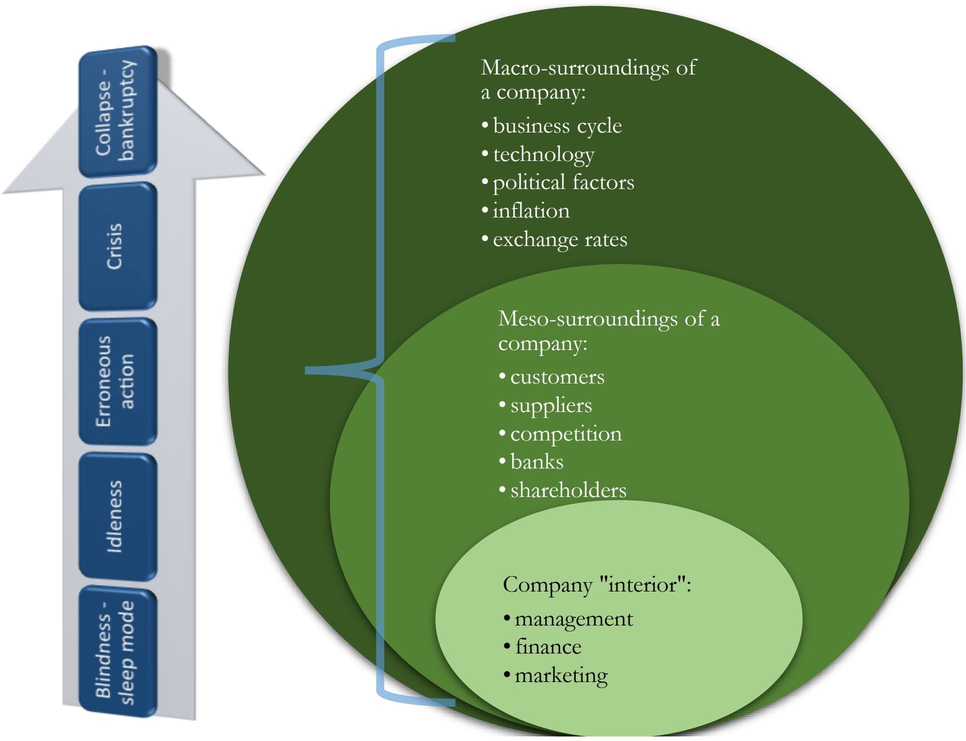


Figure 1. Phases in the process of bankruptcy and the environmental impact

Source: own study

3. METHODOLOGY

To determine the dynamic trajectories, first we had to develop forecasting models for enterprises from each analysed region of the world. Then, based on these estimated models, we determined the trajectories. The author used three methods to develop the bankruptcy-forecasting models.

The first method is multivariate discriminant analysis (MDA). Studies show it is the most popular type of model used by analysts in the prediction of bankruptcy risk (Aziz & Dar, 2006). It allows the classification of enterprises based on many explanatory variables. The method is classified as a pattern (teacher) classification, because the discriminant function's value, determined for the analysed companies, is compared to a pattern, thereby defining the firms as belonging to a class. The discriminant function is a weighted sum of the analysed financial ratios in the following form:

$$\mathbf{Z = d_0 + d_1x_1 + d_2x_2 + ... + d_nx_n} \quad (1)$$

where Z is the dependent variable, x_i is the independent financial ratios (i = 1, 2, ..., n), and d_i is the discriminant weights. When the value of function Z for an analysed firm is smaller than the cutoff value, the company is classified as a firm facing bankruptcy. However, if the value is higher than the threshold, the firm is classified as not at risk of bankruptcy.

The second most popular method is the logit model (LOG). In this study, the value of the logistic regression function is the probability of bankruptcy of the enterprise p_i:

$$\mathbf{P(Y=1) = 1 / (1 + exp^{-z}) = exp^{z} / (1 + exp^{z})} \quad (2)$$

where

P(Y=1) is the dependent variable, the probability of the financial failure of the enterprise; Z is the value of the linear function Z, where $Z = d_0 + d_1x_1 + d_2x_2 + ... + d_nx_n$ [x_i – financial ratios (i = 1, 2, ..., n); and d_i is the weights (i = 1, 2, ..., n)].

Most logit models consist of a cutoff point at 0.5. Therefore, results higher than 0.5 represent a high risk of bankruptcy (between 50% and 100% probability), and values smaller than 0.5 represent a low risk of insolvency (between 0% and 50% probability).

The first two methods are models belonging to the statistical group of methods. Although statistical methods are the most popular models in the literature, they are characterised by the use of bivalent logic (the firm is bankrupt or non-bankrupt). The company financial situation is classified as "good" when the value of the statistical function is above the estimated threshold and "bad" when it is below this value. The third developed model was the fuzzy logic model, which belongs to the soft computing group of methods. This model requires no assumptions about the learning process and is developed based on expert knowledge and experience. Such a model uses fuzzy sets to make a forecast. With fuzzy sets, we can formally define vague and ambiguous terms such as "high risk of bankruptcy", "low risk of bankruptcy", and "good financial standing". In the fuzzy logic model, an enterprise can partially belong to a set, and this belonging can be expressed by a real number from the interval [0,1]. In other words, using fuzzy logic, a company's financial situation can be assessed as partially good or partially bad.

Knowledge is a key element of the expert model. The decision-making centre of the fuzzy logic model is its basis of rules taking the form of IF-THEN, written by the author, in which expert knowledge is stored and required for an effective, factually correct interpretation of the financial ratios at the model input. The output of the model is a variable representing the forecast of the financial situation of the audited company. This variable has a value from 0 to 1, and the threshold dividing companies into those at risk and those not at risk of bankruptcy is 0.5 (variable values below 0.5 mean firms at risk of bankruptcy and those above 0.5 indicate companies not at risk of bankruptcy).

Fuzzy set A in a certain nonempty space X ($A \subseteq X$) can be defined as (Wu et al., 2010):

$$A = \{(x, \mu_A (x)) \mid x \in X \} \quad (3)$$

where μ_A: $X \rightarrow [0,1]$ is a function specifying the extent to which each element in X belongs to set A. Function μ_A is the so-called membership function of fuzzy set A. The membership function $\mu_A(x)$: $U \Rightarrow [0,1]$ is defined as follows (Korol & Fotiadis, 2016):

$$\forall_{x \in U}\ \mu_A(x) = \begin{cases} f(x), x \in X \\ 0, x \notin X \end{cases} \qquad (4)$$

where $\mu_A(x)$ is a function specifying the membership of x in set A, which is a subset of U, and f (x) is a function of values in the range [0,1]. Values of this function are called degrees of membership.

The membership function of each element $x \in X$ assigns a degree of membership to fuzzy set A, in which we can distinguish three situations:

- $\mu_A(x) = 1$ means full membership of element x in fuzzy set A,
- $\mu_A(x) = 0$ means no membership of element x in fuzzy set A, and
- $0 < \mu_A(x) < 1$ means partial membership of element x in fuzzy set A.

Membership functions are usually presented in graphical form (Nakandala et al., 2013).

When developing bankruptcy-forecasting models, variables must be selected with high predictive properties. The author of this study reviewed the literature on the financial ratios used in bankruptcy risk forecasting (e.g. Alaka et al., 2018; Altman, 2018; Delen et al., 2013; Dong et al., 2018; Jardin, 2009; Jardin, 2017; Jayasekera, 2018; Laitinen et al., 2014; Liang et al., 2016; Ooghe & Balcaen, 2006; Sun et al., 2014; Tian et al., 2015; Tian & Yu, 2017). After studying approximately 600 research papers on this subject, the author chose the 20 most popular variables recommended in the field of predicting the financial failure of enterprises (Table 1) and took into account the fact that the application of statistical models requires the chosen ratios to meet several assumptions:

- ratio values should have normal distributions,
- ratios must be independent,
- ratios must have high discriminative capacity - separating solvent from insolvent enterprises,
- observations for each object (solvent and insolvent enterprises) must be full, i.e. one must have the values of all ratios for all companies,
- the classification of companies must be clearly defined, i.e. an enterprise belonging to one group precludes its belonging to a second group.

Table 1

Financial ratios used in the study

Symbol of input variable	**Computing formula**
X1	Profit on sales / total assets
X2	(Current assets – current liabilities) / total assets
X3	(Net income + depreciation) / total liabilities
X4	Operational costs / current liabilities
X5	Stockholders’ equity / total liabilities
X6	(Stockholders’ equity + noncurrent liabilities) / fixed assets
X7	Total revenues / total assets
X8	Current assets / current liabilities
X9	Current liabilities / total assets
X10	Income before tax / current liabilities
X11	Total assets / total liabilities
X12	(Current assets – inventories) / current liabilities
X13	Income before tax / total revenues
X14	Inventories / total revenues
X15	Net income / total assets
X16	EBIT / total assets
X17	Current liabilities / stockholders’ equity
X18	Cash / total assets
X19	EBIT / interest paid
X20	Current assets / total liabilities

Source: own calculation

The following tables provide descriptive statistics for the variables used in the estimated models, divided into the three analysed regions (Table 2 - European firms, Table 3 - Asian companies and Table 4 - Latin American entities) and categorised as bankrupt or non-bankrupt. As can be seen from Tables 2, 3 and 4, for all used variables and for all enterprise regions there are significant differences between the two groups of firms, bankrupt and non-bankrupt, for such statistical measures as the mean, median, 1st quartile and 3rd quartile values.

Table 2

Statistical characteristics of the financial ratios of European enterprises (one year prior to bankruptcy)

	European enterprises							
	Mean		**Median**		**1st quartile**		**3rd quartile**	
	Bankrupt	**Non-bankrupt**	**Bankrupt**	**Non-bankrupt**	**Bankrupt**	**Non-bankrupt**	**Bankrupt**	**Non-bankrupt**
X1	-0.1154	0.0525	-0.0472	0.0387	-0.1386	0.0143	-0.0111	0.1018
X2	-1.4566	0.1462	-0.3341	0.1301	-1.9039	0.0396	0.0314	0.2298
X3	-0.1590	0.1860	-0.0652	0.1621	-0.1696	0.0484	0.0544	0.2830
X4	2.4217	4.3654	1.8600	3.0865	0.1851	1.6936	3.7183	5.0070
X5	0.2466	1.9947	0.1715	1.3848	-0.6093	0.8269	0.6369	2.0976
X6	-3.5170	1.2437	0.4078	1.1234	-2.6095	0.9335	1.0416	1.3800
X7	0.9554	0.9518	0.7962	0.8146	0.4615	0.4879	1.3416	1.3331
X8	0.7791	2.8384	0.5693	1.6475	0.1788	1.1943	1.0873	2.5610
X10	-0.2468	0.5432	-0.0872	0.1608	-0.2664	0.0804	0.0067	0.5552
X12	0.5121	1.9631	0.2311	1.1244	0.0925	0.6528	0.8926	2.2309
X13	-19.0313	0.1055	-0.1604	0.0628	-1.3252	0.0232	0.0015	0.1041
X14	0.3366	0.2902	0.1487	0.0988	0.0636	0.0628	0.1765	0.1528
X15	-0.5810	0.0306	-0.1404	0.0333	-0.6897	0.0189	0.0073	0.0675
X16	-0.3482	0.0415	-0.0938	0.0360	-0.4142	0.0166	-0.0019	0.0846
X17	2.0278	0.2507	2.9148	0.3611	-0.8206	0.6046	0.7850	0.2384
X18	0.0051	0.0196	0.0023	0.0112	0.0009	0.0065	0.0089	0.0223

Source: own calculation

Table 3

Statistical characteristics of the financial ratios of Asian enterprises (one year prior to bankruptcy)

	Asian enterprises							
	Mean		**Median**		**1st quartile**		**3rd quartile**	
	Bankrupt	**Non-bankrupt**	**Bankrupt**	**Non-bankr-upt**	**Bankrupt**	**Non-bankrupt**	**Bankrupt**	**Non-bankrupt**
X1	-0.1327	0.0619	-0.0543	0.0456	-0.1594	0.0169	-0.0127	0.1201
X2	-1.2745	0.1280	-0.2923	0.1139	-1.6659	0.0346	0.0275	0.2011
X3	-0.1501	0.1755	-0.0616	0.1530	-0.1600	0.0457	0.0513	0.2670
X4	2.9400	5.2996	2.2580	3.7470	0.2247	2.0560	4.5140	6.0785
X5	0.1627	1.3161	0.1132	0.9137	-0.4020	0.5456	0.4202	1.3840
X6	-0.4853	1.1734	0.3542	1.0600	-0.8459	0.8808	0.9042	1.3020
X7	1.1599	1.1555	0.9666	0.9889	0.5602	0.5923	1.6287	1.6184
X8	0.5668	2.0649	0.4142	1.1985	0.1301	0.8688	0.7910	1.8632
X10	-0.2838	0.6246	-0.1003	0.1850	-0.3064	0.0925	0.0077	0.6385
X12	0.3726	1.4282	0.1681	0.8180	0.0673	0.4749	0.6494	1.6230
X13	-0.2210	0.1245	-0.1892	0.0740	-0.9782	0.0274	0.0018	0.1228
X14	0.4087	0.3523	0.1806	0.1199	0.0772	0.0762	0.2143	0.1854
X15	-0.6682	0.0361	-0.1614	0.0393	-0.7931	0.0223	0.0084	0.0797
X16	-0.4004	0.0490	-0.1078	0.0425	-0.4763	0.0196	-0.0022	0.0999
X17	1.3379	0.1654	1.9232	0.2382	-0.5414	0.3989	0.5180	0.1573
X18	0.0037	0.0143	0.0017	0.0082	0.0007	0.0047	0.0065	0.0162

Source: own calculation

Table 4

Statistical characteristics of the financial ratios of Latin American firms (one year prior to bankruptcy)

	Latin enterprises							
	Mean		Median		1st quartile		3rd quartile	
	Bankrupt	Non-bankrupt	Bankrupt	Non-bankr-upt	Bankrupt	Non-bankrupt	Bankrupt	Non-bankrupt
X1	-0.1258	0.0461	-0.0514	0.0340	-0.1511	0.0126	-0.0121	0.0894
X2	-1.2236	0.1228	-0.2806	0.1093	-1.5993	0.0333	0.0264	0.1930
X3	-0.1641	0.1920	-0.0673	0.1673	-0.1750	0.0500	0.0561	0.2921
X4	2.7718	4.9965	2.1289	3.5327	0.2119	1.9384	4.2558	5.7308
X5	0.1461	1.1819	0.1016	0.8205	-0.3610	0.4900	0.3774	1.2428
X6	-0.5308	1.2835	0.3874	1.1594	-0.9253	0.9634	0.9891	1.4242
X7	1.0935	1.0894	0.9113	0.9324	0.5282	0.5584	1.5356	1.5258
X8	0.5441	1.9823	0.3976	1.1506	0.1249	0.8341	0.7594	1.7886
X10	-0.2690	0.5921	-0.0951	0.1753	-0.2904	0.0877	0.0073	0.6052
X12	0.3577	1.3710	0.1614	0.7853	0.0646	0.4559	0.6234	1.5581
X13	-0.1622	0.0928	-0.1410	0.0552	-0.8598	0.0204	0.0013	0.0915
X14	0.3853	0.3322	0.1702	0.1130	0.0728	0.0718	0.2020	0.1748
X15	-0.6333	0.0269	-0.1530	0.0293	-0.7517	0.0166	0.0080	0.0593
X16	-0.3795	0.0365	-0.1022	0.0316	-0.4514	0.0146	-0.0021	0.0744
X17	1.2015	0.1485	1.7270	0.2139	-0.4862	0.3583	0.4651	0.1412
X18	0.0036	0.0137	0.0016	0.0079	0.0006	0.0046	0.0062	0.0156

Source: own calculation

The research relies on six enterprise samples. Three samples are learning samples, and three are testing samples. The learning and testing samples comprise enterprises from three different regions of the world: Latin America (Mexico, Argentina, Chile, Peru, Brazil and Colombia), Europe (Germany, France, Spain, Finland, Italy, Poland and Sweden), and Asia (Taiwan, Japan, China, South Korea and Malaysia). Each testing sample consists of 300 bankrupt and 300 non-bankrupt enterprises, while each learning sample consists of 50 bankrupt and 50 non-bankrupt firms. The fact that an enterprise is in a good financial condition was assumed based on the overall analysis of financial statements. In the assessment, profitability, liquidity and debt ratios were mainly considered. The companies were selected for which there was no doubt that they are not at risk of failing. The enterprises at risk of bankruptcy were chosen based on the following three criteria:

- information from the firm's authorities about the threat of collapse,
- court judgments declaring bankruptcy, and
- liquidation of the company.

The chosen enterprises were companies in the service and manufacturing sectors. This study omitted enterprises from the financial sector (banks and insurance companies) due to the very different characteristics of these types of enterprises. The main difficulty to overcome was finding 350 enterprises for each investigated region that were at risk of bankruptcy and for which there were available data for such a long analytical horizon prior to financial crisis. The forecasting horizon for all enterprises and all models (the information was taken from the years 1995–2020, depending on the enterprise) comprises fifteen years prior to bankruptcy (data for each year separately).

Apart from developing long-term trajectories, the author also evaluated the short- and long-term effectiveness of the estimated models and identified which model characterises the forecast with the smallest decrease in effectiveness in regard to the increasing horizon for each region of the world separately. The following formula was used to calculate overall effectiveness:

$$S=\{1-[(D1+D2)/(BR+NBR)]\}*100\% \quad (5)$$

where D1 is the number of insolvent enterprises classified by the model as solvent, D2 is the number of solvent companies classified by the model as insolvent, BR is the number of distressed enterprises in the sample, and NBR is the number of "healthy" firms in the sample.

4. EMPIRICAL RESULTS AND DISCUSSION

To determine enterprise trajectories, nine forecasting models have been constructed: three models of multivariate discriminant analysis, three logit models and three fuzzy logic models (one for each region of the world) with the use of learning samples.

The bankruptcy-prediction models for Europe mainly consist of three variables: profit on sales / total assets (X_1), (current assets - current liabilities) / total assets (X_2), and (net income + depreciation) / total liabilities (X_3). Other ratios used in these models are X_4, X_5, X_6, X_7, X_8 and X_{13} (the formulas are given in Table 1). For the Latin American models, the significant variables were profit on sales / total assets (X_1), operational costs / current liabilities (X_4), and (stockholders' equity + noncurrent liabilities) / fixed assets (X_6). In addition to these variables, the following ratios were also implemented in the models: X_3, X_7, X_8, X_{12}, X_{13} and X_{16}. For the models constructed for Asian firms, the significant variables refer to income before tax / current liabilities (X_{10}) and cash / total assets (X_{18}). Other ratios used in these predicting models are X_1, X_2, X_3, X_6, X_8, X_{12}, X_{17} and X_{14}.

The first estimated model was the multivariate discriminant analysis model. The estimated model differs from most models of discriminant analysis found in the literature both because it consists of two functions in the model and in the way it classifies companies into those at risk and not at risk of bankruptcy. Depending on the higher value of Z_{BAN} or Z_{NON}, a company is classified into one of the populations (non-bankrupt, bankrupt) of companies. Thus, if $Z_{BAN} > Z_{NON}$, the company is considered to be at risk of bankruptcy; when $Z_{BAN} < Z_{NON}$, the risk does not exist, and the company's financial situation is assessed positively.

Using a forward stepwise regression method, the following forms of the MDA models were estimated:

- for European enterprises:

$$\mathbf{Z_{ban} = -2.98 - 2.71 * X_1 - 6.65 * X_2 - 0.67 * X_3 + 1.9 * X_6 + 0.89 * X_7}$$
$$\mathbf{Z_{non} = -2.99 + 7.12 * X_1 + 1.15 * X_2 + 2.21 * X_3 + 0.51 * X_6 + 2.0 * X_7}$$

- for Asian enterprises:

$$\mathbf{Z_{ban} = -10.23 + 2.59 * X_6 - 3.15 * X_8 - 9.16 * X_{10} + 0.19 * X_{14} - 0.021 * X_{18}}$$
$$\mathbf{Z_{non} = -10.72 + 1.32 * X_6 + 4.89 * X_8 + 6.45 * X_{10} + 1.73 * X_{14} + 0.192 * X_{18}}$$

- for Latin American enterprises:

$$\mathbf{Z_{ban} = -4.15 - 1.25 * X_1 - 2.19 * X_3 - 3.95 * X_4 + 1.9 * X_6 + 1.29 * X_{16}}$$
$$\mathbf{Z_{non} = -4.59 + 9.37 * X_1 + 4.17 * X_3 + 0.55 * X_4 + 0.51 * X_6 + 3.32 * X_{16}}$$

All three models comprise five financial ratios, but for Asian enterprises, the model contains very different ratios than is the case for European and Latin American firms. The models for European and Latin American companies share three of the same ratios (with different coefficients): X_1, X_3 and X_6.

In the second step, the logit model was estimated. This model was also developed using forward stepwise regression. For individual samples of enterprises, the following forms of this model have been estimated:

- for European firms:

$$\mathbf{Z = 2.0 - 10.19 * X_1 - 4.58 * X_3 - 0.57 * X_4}$$

- for Asian firms:

$$\mathbf{Z = -1.12 - 0.29 * X_2 - 7.21 * X_{15} - 1.83 * X_{18}}$$

- for Latin American firms:

$$\mathbf{Z = 1.25 - 12.05 * X_1 - 0.87 * X_4 - 2.78 * X_{12}}$$

The last model developed was the fuzzy logic model. The ratios used for fuzzy logic models are based on a correlation matrix by choosing only those features that are poorly correlated with each other and strongly correlated with the grouping variable, representing information about the threat of bankruptcy or the lack of risk of bankruptcy. This approach ensured the selection of such features which do not duplicate information provided by other financial ratios, while being good representatives of the ratios not selected as diagnostic. For each chosen financial ratio, the author defined two fuzzy sets, BAD ("low") and GOOD ("high"), and their membership functions. The fuzzy sets and the shape of membership functions have been arbitrarily set by the author. The assessment ratio (as "good" or "bad") is based on a statistical analysis. The author counted the value of the first and third quartile for each financial ratio separately for companies in good financial condition and for companies at risk of bankruptcy for all fifteen years prior to bankruptcy. The third-quartile value three years prior to bankruptcy for bankrupt companies serves as a cutoff point (the index is recognised as "bad" below the cutoff value). These values are shown in Table 5.

Table 5

The cutoff values of input data in the fuzzy logic model

Symbol of financial ratio	Cutoff value in the model
For European companies	
$\mathbf{X_1}$	0.015
$\mathbf{X_2}$	0.11
$\mathbf{X_5}$	0.75
$\mathbf{X_8}$	1.05
$\mathbf{X_{13}}$	0.01
For Asian firms	
$\mathbf{X_1}$	0.023
$\mathbf{X_3}$	0.153
$\mathbf{X_{10}}$	0.214
$\mathbf{X_{12}}$	0.65
$\mathbf{X_{17}}$	0.8
For Latin American enterprises	
$\mathbf{X_1}$	0.028
$\mathbf{X_6}$	0.87
$\mathbf{X_7}$	0.475
$\mathbf{X_8}$	1.12
$\mathbf{X_{13}}$	0.035

Source: own calculation

Figure 2 presents an example of membership functions for ratio X1 in the analysis of companies from Asia, and Figure 3 shows the example for ratio X8 in the analysis of companies from Europe.

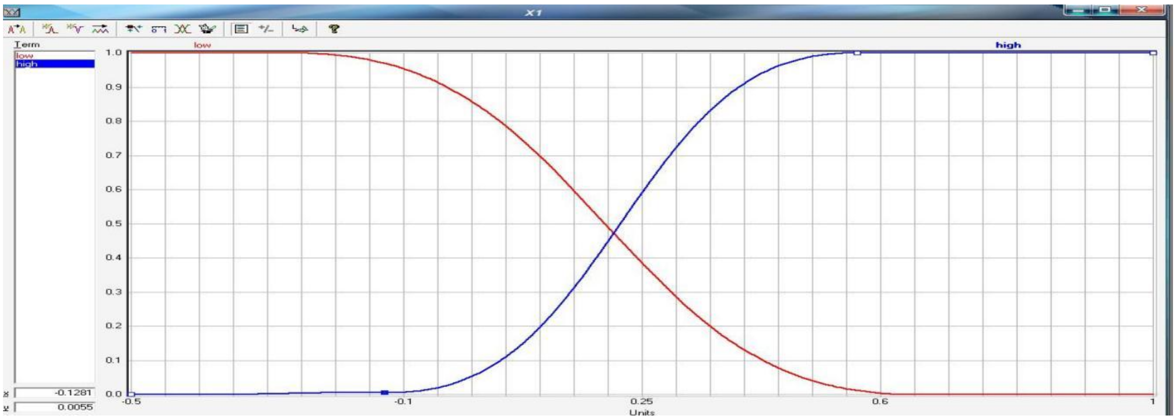


Figure 2. The membership functions for ratio X1 – profit on sales / total assets (Asian firms)

Source: own calculation

For ratio X1, representing the profitability of total assets, the cutoff value between a negative and a positive scenario is 0.023. Results lower than -0.03 are fully negative and belong to the membership function "LOW". Results of this ratio higher than 0.06 are fully positive, which means they belong to the membership function "HIGH". The results of ratio X1 between -0.03 to 0.06 belong to both fuzzy subsets with different values of membership functions. For example, when ratio X1 equals 0.023, the value of the function of membership in the "LOW" set is 0.5 and for the "HIGH" set is 0.5.

Using such defined membership functions, a certain ratio value can be "partially good" and "partially bad." Classical logic offers no such possibility because it is bivalent, so the value of the ratio is "good" or "bad." That is why using classical logic to forecast the financial position of companies negatively affects the quality of the models.

Figure 3. The membership functions for ratio X8 – current assets / current liabilities (European enterprises)

Source: own calculation

For each sample of enterprises, the author has developed 25 decision-making "IF-THEN" rules, presented in Table 6.

Table 6

The developed decision rules of fuzzy set models

European enterprises						
No.	**If X_1 is**	**If X_2 is**	**If X_5 is**	**If X_8 is**	**If X_{13} is**	**Then output is**
1.	<= 0.015	<= 0.11	<= 0.75	<= 1.05	<= 0.01	0
2.	<= 0.015	<= 0.11	<= 0.75	<= 1.05	> 0.01	0
3.	<= 0.015	<= 0.11	<= 0.75	> 1.05	> 0.01	0
4.	<= 0.015	<= 0.11	> 0.75	> 1.05	> 0.01	1
5.	<= 0.015	> 0.11	> 0.75	> 1.05	> 0.01	1
6.	> 0.015	> 0.11	> 0.75	> 1.05	> 0.01	1
7.	<= 0.015	<= 0.11	> 0.75	<= 1.05	> 0.01	0
8.	<= 0.015	> 0.11	<= 0.75	<= 1.05	> 0.01	0
9.	> 0.015	<= 0.11	<= 0.75	<= 1.05	> 0.01	0
10.	<= 0.015	<= 0.11	<= 0.75	> 1.05	<= 0.01	0
11.	<= 0.015	<= 0.11	> 0.75	<= 1.05	<= 0.01	0
12.	<= 0.015	> 0.11	<= 0.75	<= 1.05	<= 0.01	0
13.	> 0.015	<= 0.11	<= 0.75	<= 1.05	<= 0.01	0
14.	<= 0.015	> 0.11	> 0.75	> 1.05	<= 0.01	1
15.	<= 0.015	<= 0.11	> 0.75	> 1.05	<= 0.01	0
16.	<= 0.015	> 0.11	<= 0.75	> 1.05	<= 0.01	0
17.	<= 0.015	> 0.11	> 0.75	<= 1.05	<= 0.01	0
18.	> 0.015	<= 0.11	> 0.75	<= 1.05	<= 0.01	0
19.	> 0.015	<= 0.11	<= 0.75	> 1.05	<= 0.01	0
20.	> 0.015	<= 0.11	<= 0.75	> 1.05	> 0.01	1
21.	> 0.015	> 0.11	<= 0.75	> 1.05	> 0.01	1
22.	> 0.015	> 0.11	<= 0.75	<= 1.05	> 0.01	1
23.	> 0.015	> 0.11	> 0.75	<= 1.05	<= 0.01	1
24.	> 0.015	> 0.11	<= 0.75	> 1.05	<= 0.01	1
25.	> 0.015	<= 0.11	> 0.75	> 1.05	> 0.01	1
Asian enterprises						
No.	**If X_1 is**	**If X_3 is**	**If X_{10} is**	**If X_{12} is**	**If X_{17} is**	**Then output is**
1.	<= 0.023	<= 0.153	<= 0.214	<= 0.65	<= 0.8	0
2.	<= 0.023	<= 0.153	<= 0.214	<= 0.65	> 0.8	0
3.	<= 0.023	<= 0.153	<= 0.214	> 0.65	> 0.8	0
4.	<= 0.023	<= 0.153	> 0.214	> 0.65	> 0.8	1
5.	<= 0.023	> 0.153	> 0.214	> 0.65	> 0.8	1
6.	> 0.023	> 0.153	> 0.214	> 0.65	> 0.8	1
7.	<= 0.023	<= 0.153	> 0.214	<= 0.65	> 0.8	0
8.	<= 0.023	> 0.153	<= 0.214	<= 0.65	> 0.8	0
9.	> 0.023	<= 0.153	<= 0.214	<= 0.65	> 0.8	0
10.	<= 0.023	<= 0.153	<= 0.214	> 0.65	<= 0.8	0
11.	<= 0.023	<= 0.153	> 0.214	<= 0.65	<= 0.8	0
12.	<= 0.023	> 0.153	<= 0.214	<= 0.65	<= 0.8	0
13.	> 0.023	<= 0.153	<= 0.214	<= 0.65	<= 0.8	0
14.	<= 0.023	> 0.153	> 0.214	> 0.65	<= 0.8	1
15.	<= 0.023	<= 0.153	> 0.214	> 0.65	<= 0.8	0
16.	<= 0.023	> 0.153	<= 0.214	> 0.65	<= 0.8	0
17.	<= 0.023	> 0.153	> 0.214	<= 0.65	<= 0.8	0
18.	> 0.023	<= 0.153	> 0.214	<= 0.65	<= 0.8	0
19.	> 0.023	<= 0.153	<= 0.214	> 0.65	<= 0.8	0
20.	> 0.023	<= 0.153	<= 0.214	> 0.65	> 0.8	1
21.	> 0.023	> 0.153	<= 0.214	> 0.65	> 0.8	1
22.	> 0.023	> 0.153	<= 0.214	<= 0.65	> 0.8	1

23.	> 0.023	> 0.153	> 0.214	<= 0.65	<= 0.8	1
24.	> 0.023	> 0.153	<= 0.214	> 0.65	<= 0.8	1
25.	> 0.023	<= 0.153	> 0.214	> 0.65	> 0.8	1
Latin American enterprises						
No.	**If X_1 is**	**If X_6 is**	**If X_7 is**	**If X_8 is**	**If X_{13} is**	**Then output is**
1.	<= 0.028	<= 0.87	<= 0.475	<= 1.12	<= 0.035	0
2.	<= 0.028	<= 0.87	<= 0.475	<= 1.12	> 0.035	0
3.	<= 0.028	<= 0.87	<= 0.475	> 1.12	> 0.035	0
4.	<= 0.028	<= 0.87	> 0.475	> 1.12	> 0.035	1
5.	<= 0.028	> 0.87	> 0.475	> 1.12	> 0.035	1
6.	> 0.028	> 0.87	> 0.475	> 1.12	> 0.035	1
7.	<= 0.028	<= 0.87	> 0.475	<= 1.12	> 0.035	0
8.	<= 0.028	> 0.87	<= 0.475	<= 1.12	> 0.035	0
9.	> 0.028	<= 0.87	<= 0.475	<= 1.12	> 0.035	0
10.	<= 0.028	<= 0.87	<= 0.475	> 1.12	<= 0.035	0
11.	<= 0.028	<= 0.87	> 0.475	<= 1.12	<= 0.035	0
12.	<= 0.028	> 0.87	<= 0.475	<= 1.12	<= 0.035	0
13.	> 0.028	<= 0.87	<= 0.475	<= 1.12	<= 0.035	0
14.	<= 0.028	> 0.87	> 0.475	> 1.12	<= 0.035	1
15.	<= 0.028	<= 0.87	> 0.475	> 1.12	<= 0.035	0
16.	<= 0.028	> 0.87	<= 0.475	> 1.12	<= 0.035	0
17.	<= 0.028	> 0.87	> 0.475	<= 1.12	<= 0.035	0
18.	> 0.028	<= 0.87	> 0.475	<= 1.12	<= 0.035	0
19.	> 0.028	<= 0.87	<= 0.475	> 1.12	<= 0.035	0
20.	> 0.028	<= 0.87	<= 0.475	> 1.12	> 0.035	1
21.	> 0.028	> 0.87	<= 0.475	> 1.12	> 0.035	1
22.	> 0.028	> 0.87	<= 0.475	<= 1.12	> 0.035	1
23.	> 0.028	> 0.87	> 0.475	<= 1.12	<= 0.035	1
24.	> 0.028	> 0.87	<= 0.475	> 1.12	<= 0.035	1
25.	> 0.028	<= 0.87	> 0.475	> 1.12	> 0.035	1

Source: own calculation

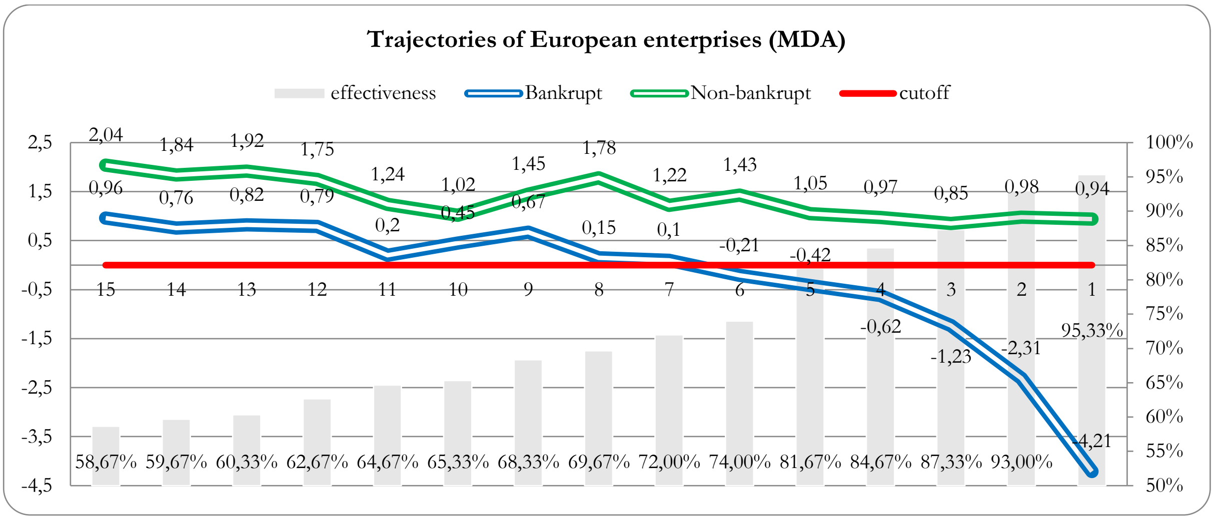
The next step in developing the trajectories of the enterprises was to calculate the results of the nine created forecasting models (three models for each region) for all enterprises from the testing sample for each analysed year in the 15-year horizon (81 000 results). In the subsequent step, the testing sample was divided into 300 bankrupt and 300 non-bankrupt enterprises. Then, the median of values generated by the forecasting models was calculated separately for these two types of companies for all the years (Figures 4, 5 and 6). In the last stage, the author calculated the overall effectiveness of each model for all the years (Table 7).

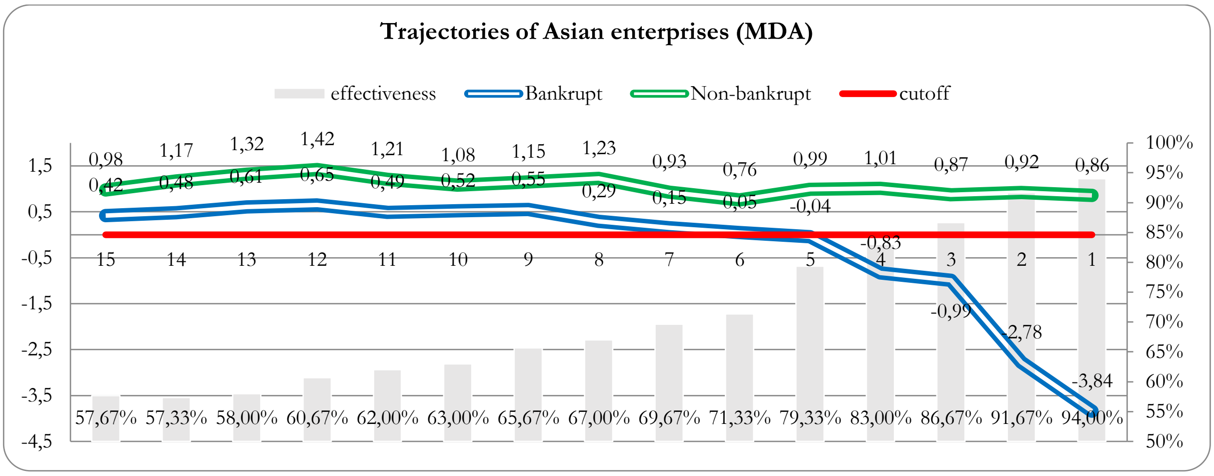
Looking at the trajectories of non-bankrupt and bankrupt enterprises determined with the use of multivariate discriminant analysis models (Figure 4), **we can draw three important conclusions**:

- For all 15 years analysed in all three regions, **the values of the trajectories of non-bankrupt enterprises were at least twice as high as the values of the trajectories of bankrupt firms**. This result means that the determined trajectories clearly present significant differences in development between non-bankrupt companies and future insolvent enterprises.
- In the case of European enterprises, **the bankrupt trajectory noted negative values (below the cutoff point) in the 6th year prior to bankruptcy. In the case of Asian and Latin American companies, the bankrupt trajectories recorded negative values during the 5th and 4th years, respectively, before financial failure**.

- The trajectories of non-bankrupt firms in all three regions were characterised by stability. Although the values fluctuate, the result is natural as the economic situation was also changing, but fluctuations were never drastic and were kept within specific boundaries **much above the symptoms of insolvency generated by future bankrupt enterprises** (the trajectory of bankrupt firms).

Further analysis (Figure 5) shows the logit models are characterised by weaker bankruptcy forecasting properties. The effectiveness of the models is lower than the effectiveness of the multivariate discriminant analysis models. Additionally, the developed trajectories show significant symptoms of bankruptcy at a later phase. In the case of European firms, the values of the bankrupt trajectory crossed the cutoff point (0.5) in the 5th year, and for both Latin American and Asian enterprises the values crossed in the 4th year prior to bankruptcy. However, **for all three regions, the trajectory of bankrupt firms was evident and significantly different than the trajectory of non-bankrupt entities**. The values of these trajectories for the whole 15-year period were at least twice as low as the values of the trajectories of "healthy" companies.





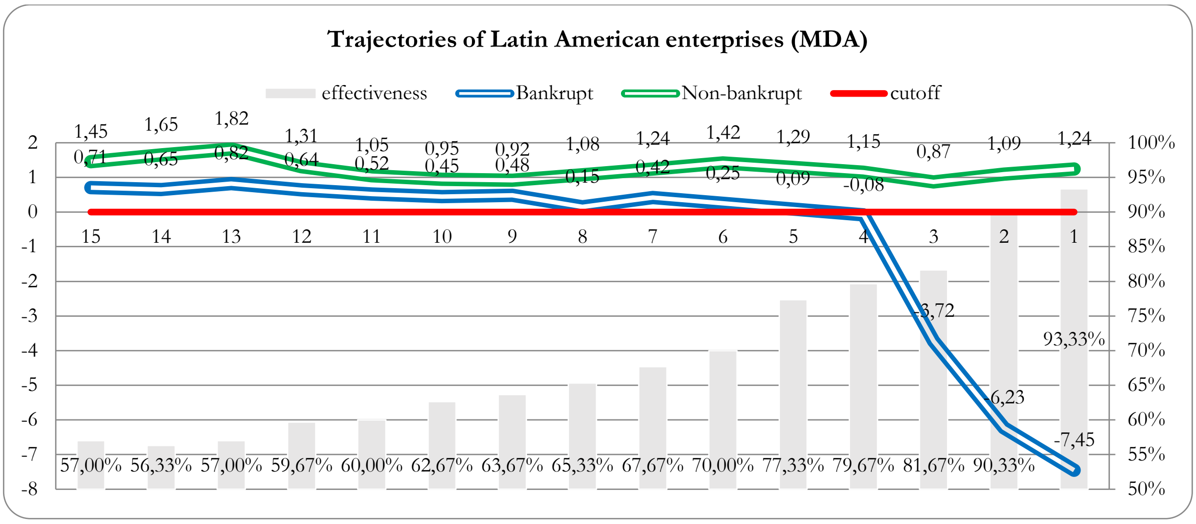
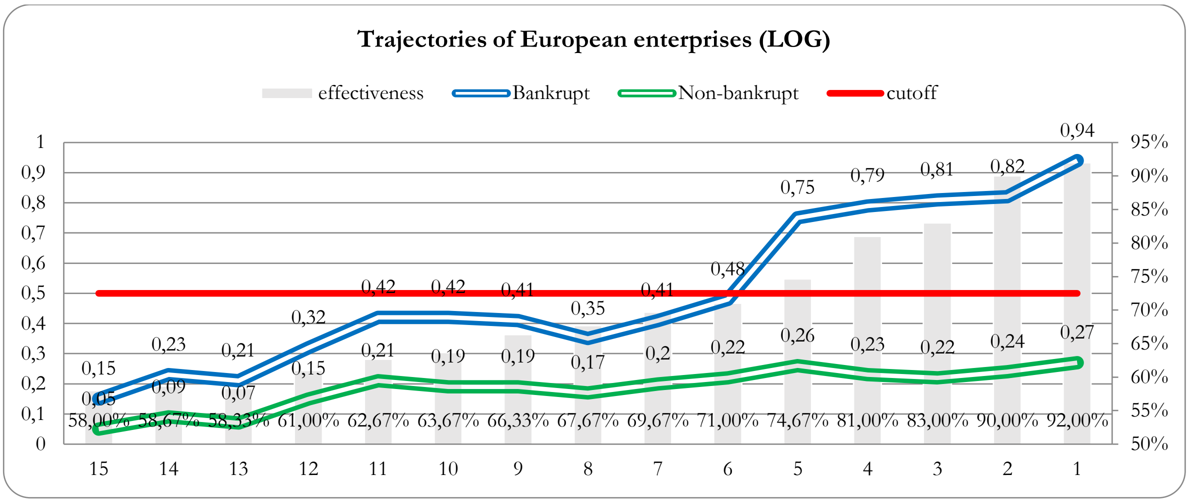
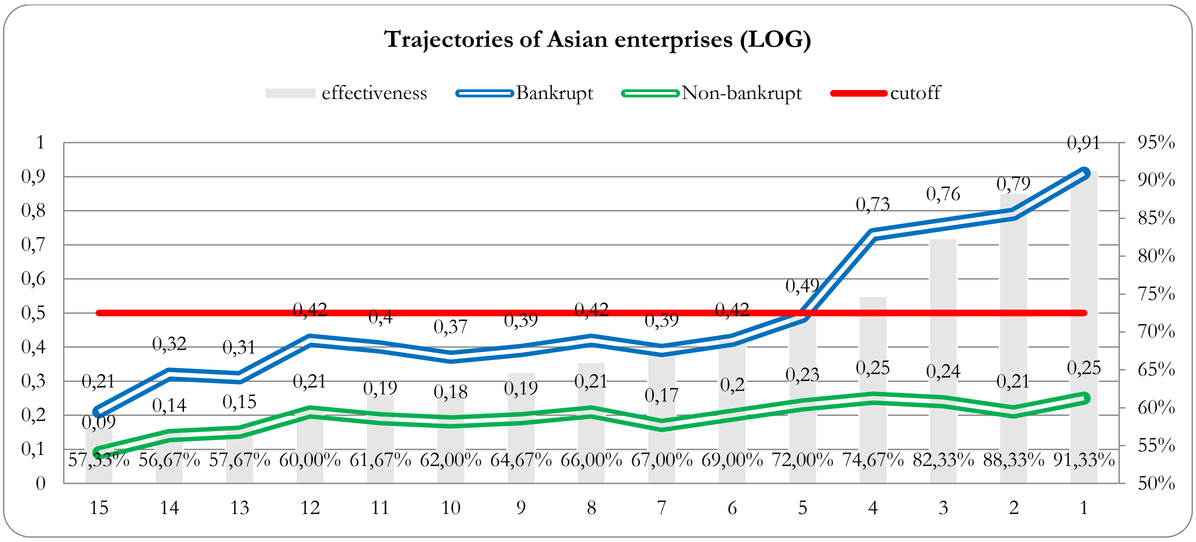


Figure 4. Trajectories of non-bankrupt and bankrupt enterprises based on MDA models

Source: own calculation





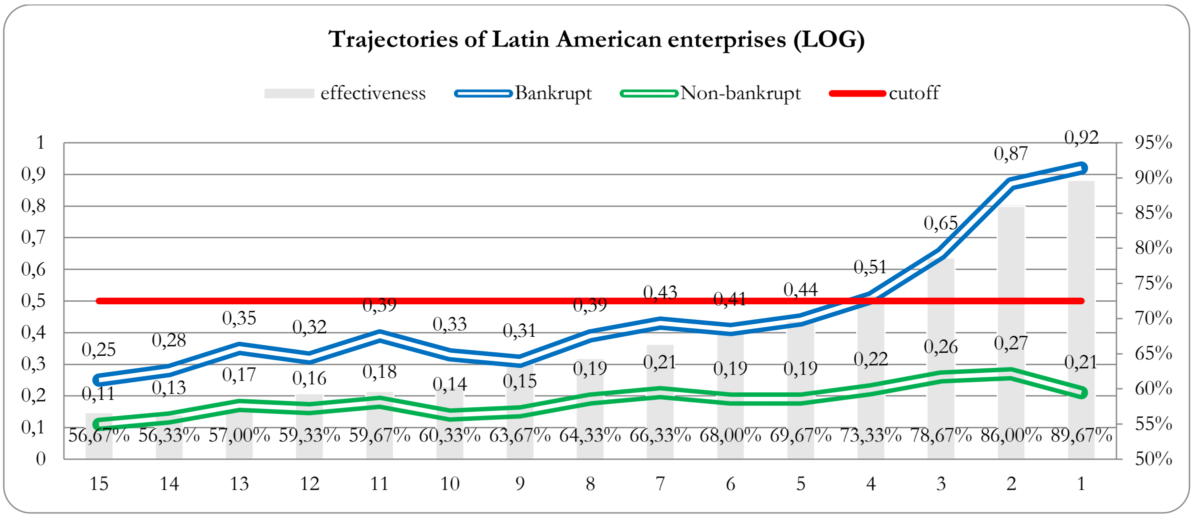
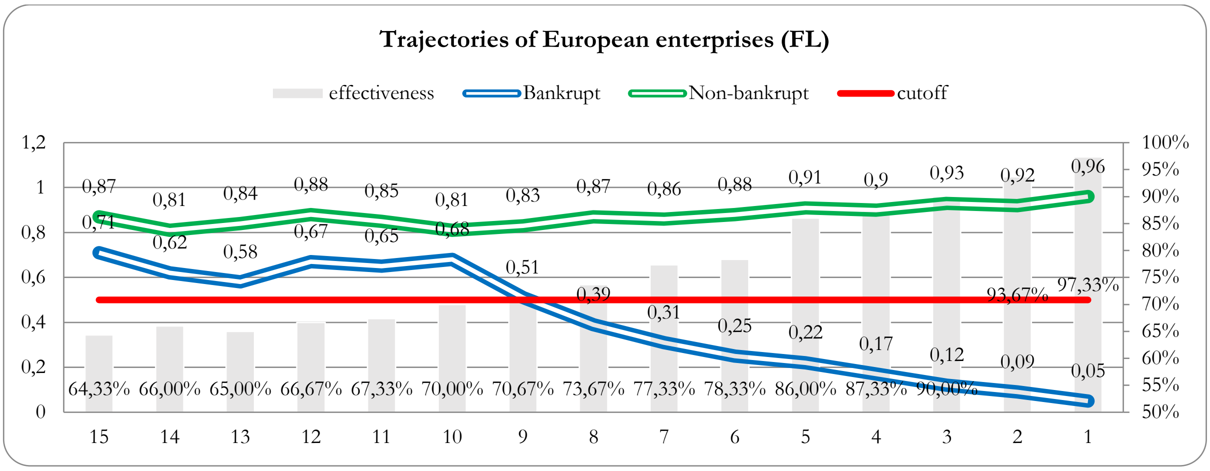
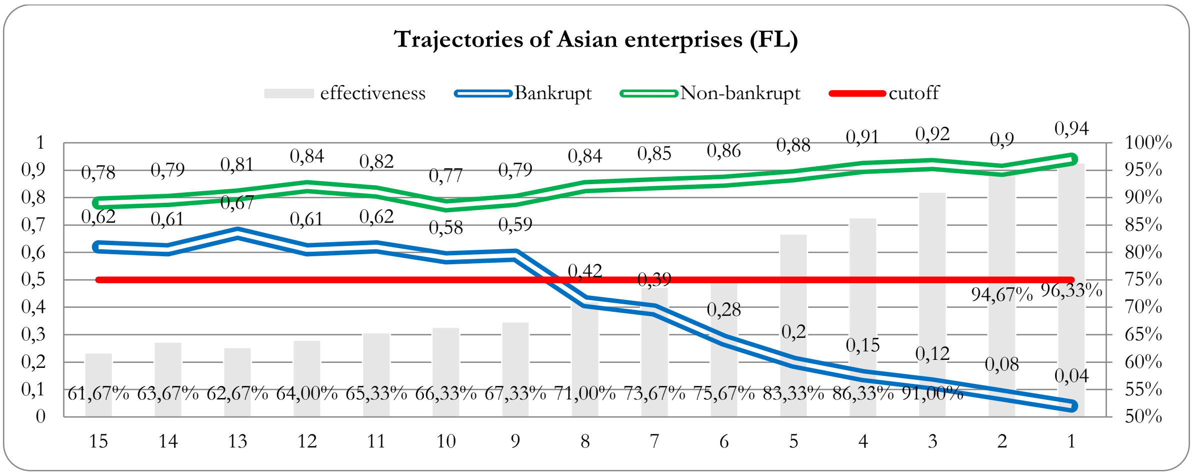


Figure 5. Trajectories of non-bankrupt and bankrupt enterprises based on the LOG models

Source: own calculation

The last type of forecasting model used, the **fuzzy logic model, proved its superiority over the first two implemented methods, both in effectiveness and in the determined trajectories**. From Figure 6, we can observe that the bankrupt trajectory of firms from all three regions clearly presents a systematic increase in the risk of financial failure for the studied entities. A gradual decrease in solvency capability **can be seen already in the 8th year before bankruptcy**. Using the FL model, the bankrupt trajectory for up to 8 years of analysis is below the cutoff point of 0.5 (when the value is lower, the bankruptcy risk is higher). **The fuzzy logic models also proved the non-bankrupt trajectory is significantly different from the bankrupt trajectory in the long term for all 15 years of analysis**.





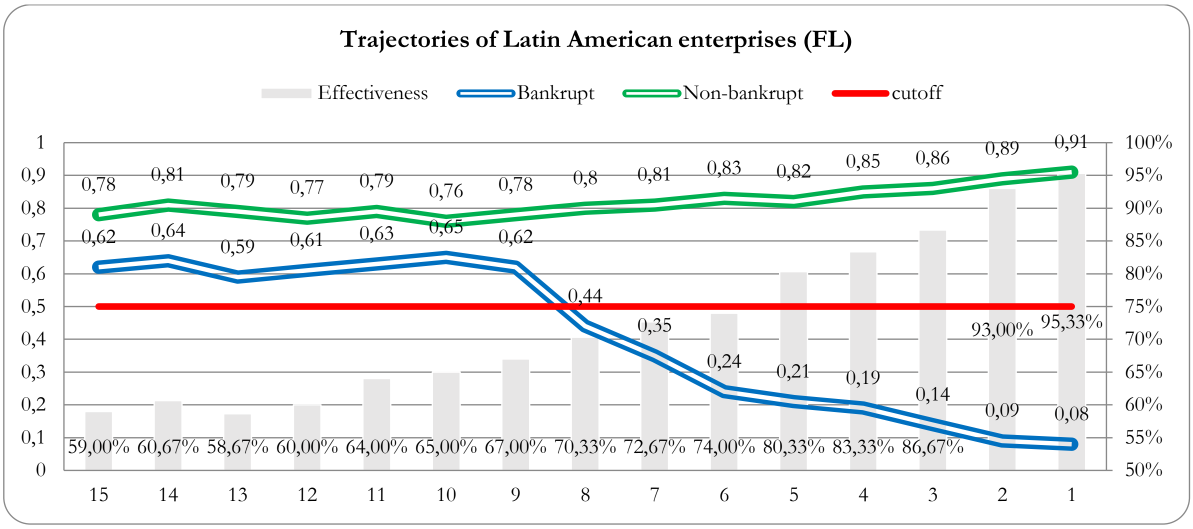


Figure 6. Trajectories of non-bankrupt and bankrupt enterprises based on the FL models

Source: own calculation

An examination of the effectiveness of the models (Table 6) shows all of them stand out with good results in the forecasting horizon of one, two and three years, with an effectiveness above 80%. The results for European firms show the highest effectiveness was achieved using the fuzzy logic model, with 97.33% correct classifications one year before bankruptcy, 93.67% correct classifications two years prior to financial failure and 90.00% correct classifications three years before bankruptcy. In the case of Asian firms, the fuzzy logic model is again characterised by the highest effectiveness (96.33% one year, 94.67% two years and 91.00% three years before bankruptcy). Similarly, in the case of Latin American enterprises, the fuzzy logic model recorded the best forecasting properties, with an effectiveness of 95.33% one year, 93.00% two years and 86.67% three years prior to financial failure.

In a horizon longer than 5 years before bankruptcy, the multivariate discriminant analysis and logit models generated an effectiveness lower than 80% for enterprises from all regions (an exception is the MDA model for European firms, with effectiveness of 81.67%). Moreover, in a forecasting horizon exceeding 7 years prior to financial failure, the noted effectiveness of these models is lower than 70%. Generally, effectiveness below 70% is recognised as low. **Meanwhile, the fuzzy logic model maintained an effectiveness level above 70% until the 10th year for European firms and the 8th year for Asian and Latin American entities**. Additionally, the fuzzy logic model outperforms the multivariate discriminant analysis and logit models across the entire forecasting horizon of 15 years for all three regions (Table 6).

Table 7

Effectiveness of the forecasting models

Years before bankruptcy	European			Asian			Latin American		
	MDA	LOG	FL	MDA	LOG	FL	MDA	LOG	FL
1	95.33	92.00	**97.33**	94.00	91.33	**96.33**	93.33	89.67	**95.33**
2	93.00	90.00	**93.67**	91.67	88.33	**94.67**	90.33	86.00	**93.00**
3	87.33	83.00	**90.00**	86.67	82.33	**91.00**	81.67	78.67	**86.67**
4	84.67	81.00	**87.33**	83.00	74.67	**86.33**	79.67	73.33	**83.33**
5	81.67	74.67	**86.00**	79.33	72.00	**83.33**	77.33	69.67	**80.33**
6	74.00	71.00	**78.33**	71.33	69.00	**75.67**	70.00	68.00	**74.00**
7	72.00	69.67	**77.33**	69.67	67.00	**73.67**	67.67	66.33	**72.67**
8	69.67	67.67	**73.67**	67.00	66.00	**71.00**	65.33	64.33	**70.33**
9	68.33	66.33	**70.67**	65.67	64.67	**67.33**	63.67	63.67	**67.00**
10	65.33	63.67	**70.00**	63.00	62.00	**66.33**	62.67	60.33	**65.00**
11	64.67	62.67	**67.33**	62.00	61.67	**65.33**	60.00	59.67	**64.00**
12	62.67	61.00	**66.67**	60.67	60.00	**64.00**	59.67	59.33	**60.00**
13	60.33	58.33	**65.00**	58.00	57.67	**62.67**	57.00	57.00	**58.67**
14	59.67	58.67	**66.00**	57.33	56.67	**63.67**	56.33	56.33	**60.67**
15	58.67	58.00	**64.33**	57.67	57.33	**61.67**	57.00	56.67	**59.00**

Source: own calculation

Results between the regions indicate models for Latin American firms show a relatively higher decrease in forecasting performance, which could be caused by the higher complexity of markets and of the forecasting process itself.

5. CONCLUSION

This study is one of the first attempts in the literature to forecast corporate bankruptcy risk in a 15-year horizon. The majority of studies are limited to a two to three-year period. It presents a multiregional investigation of the bankruptcy process with a determination of dynamic systems in the form of 15-year trajectories. The results indicate that the traditional bankruptcy-prediction models (MDA and LOG) achieve short-term performance (for up to three years prior to financial failure) at a high level. However, when the prediction horizon exceeds three years, their effectiveness drastically decreases. In this paper, we showed how to improve the effectiveness of forecasting corporate distress risk in both short and long horizons, exceeding five years prior to the announcement of bankruptcy. Thus, our empirical study presents novelties with respect to the existing literature and reveals the following conclusions:

1. **Significant, large differences exist in development between non-bankrupt enterprises and future bankrupt firms.** Most of those distressed firms were not in danger of bankruptcy 10 or 15 years before such risk occurred. **However, this research proved the process of going bankrupt is very long,** and **by** using the created models and trajectories, analysts can and should identify the symptoms of going bankrupt long before the real bankruptcy risk occurs.
2. **The forecasting models have been underestimated.** The models are characterised by high effectiveness in a horizon forecast of one to three years before financial failure. Prolonging the forecast period, we can observe that the effectiveness of all models very much decreases. However, these studies showed that in spite of the decrease in effectiveness, with the use of trajectories we can identify early warning symptoms much sooner than we could previously. **For all three regions, the trajectory of**

bankrupt firms was evident and significantly different than the trajectory of non-bankrupt entities. The values of these trajectories for the whole 15-year period were at least twice as low as the values of the trajectories of "healthy" companies. In this context, we can state the models are efficient at differentiating good enterprises from firms at risk of financial failure across the entire horizon of 15 years of analysis.

3. Although the forecasting models for European and Latin American enterprises very often use the same type of financial ratios, meaning similar financial information is necessary to evaluate the process of forecasting the risk of financial failure, **the models for European firms are much more stable, so the forecasting horizon for these firms can be increased compared with that for Latin American companies. This result** can be a sign that Latin American enterprises operate in more dynamic and unpredictable environments and markets.
4. **The fuzzy logic models outperform** the statistical forecasting models both in effectiveness and in the ability to determine the trajectories for bankrupt and non-bankrupt firms. In all three samples of enterprises, the most effective models are the fuzzy logic models in both the short- and long-term forecasting horizon, showing the smallest decrease in effectiveness in regard to an increase in the forecasting horizon.

The study showed that implementing financial data science in the form of fuzzy logic can improve the quality of forecasts. The multidisciplinary research approach also demonstrated superiority over traditional forecasting models in terms of a much longer forecasting horizon.

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