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# Model of world technological and economic efficiency frontiers

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**Abstract**. The model generalizes the classical concept of the world's technological frontier. The article is devoted to the theory of comparative analysis of the efficiency of national economies using the non-parametric method of data shell analysis (DEA). The proposed model summarizes the classical concept of the world's technological frontier. Algebraically, it is the solution to the well-known linear programming problem, which determines the countries that use the best combinations of capital and labour to produce a unit of output. Geometrically, it is the envelopment of the states of the studied countries in the plane of relative volumes of factors of production ("technological plane"). Authors add to this problem three other problems and the classical equation of nominal national income distribution between the factors of production. The first of these problems identifies countries that have the best combinations of real labour and capital prices. Analogously, the geometric solution to this problem authors defined the world economic frontier. It is the envelopment of the states of countries in the plane of real prices of production factors ("economic plane"). The other two frontiers consist of countries with the best combinations of parameters of the national function of product price distribution and product volume distributions between labour and capital. The economic frontier, built by the envelopment method of the price distribution function, can be depicted on the technological plane as an envelopment of the worst technological conditions. Similarly, the technological frontier, built by the envelopment method of product distribution functions, can be depicted on the economic plane as the shell of the worst economic conditions. Approbation of the model on data from 13 countries

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in 2017 revealed a geographical axis in the direction of "northwest-southeast" on the European continent.

**Keywords:** data envelopment analysis, global technological frontier, the efficiency of national economies, theory of production factors, net output, average value-added, factorization of export-import flows

**JEL Classification:** C14, C61, F50, O11, O57

## 1. INTRODUCTION

A nonparametric approach to economic data processing has recently become increasingly popular. The Data Envelopment Analysis (DEA) is the implementation of this approach. The application of the DEA method in statistical data processing can be explained by the increasing complexity and, sometimes, inability to build adequate econometric models. Initially, the DEA was used at the micro level – to plot the efficiency frontiers of a group of compared subjects.

Since the end of the twentieth century, this analysis has also been used at the macro level – to compare the national economies of different countries. In most common models, the leading countries with the best combinations of technological efficiency indicators (capital and labour productivity) form a World Technology Frontier (WTF). Other countries are considered outsiders relative to this frontier. In an alternative WTF model, instead of the envelopment of combinations of indicators (country points), the envelopment of indicator functions (national efficiency frontiers) is constructed (Caselli & Coleman II, 2000).

However, the goal of producers is not only to increase technological efficiency, which is expressed in reducing capital and labour intensity (K/Y, L/Y), but also to increase economic (in a narrow sense) efficiency – to reduce the real prices of the production factors used. These two pairs of goals – reducing the intensity of use of production factors and reducing real rental and wage rates – are competing interests, as reducing the first pair of indicators increases the other two and vice versa.

Thus, the subject of the proposed study is an adequate presentation of the effectiveness of the interdependence of these competing goals in the theory of frontiers.

## 2. LITERATURE REVIEW

It is possible to accentuate the books, written by the founder of the modern DEA Cooper et al., among the works devoted to the mathematical foundations of DEA (2006, 2007, 2011). The subject of these books is the problems of constructing various efficiency frontiers, their geometric interpretation, correct algebraic formulation of linear programming problems and their solution. The main models of DEA are described in the book by an international team of authors (Ed. by Charles & Kumar, 2012) and examples of its practical application are given. Various aspects of applying statistical methods in Data Envelopment Analysis are considered by Sickles & Zelenyuk (2019).

At the same time, certain precautions about the use of DEA in practice can be found in the theoretical literature. Thus, Emiruznejad & Cabanda (2013) conclude that the DEA findings have to be thoroughly explained to prevent the emergence of any incorrect signals and incorrect recommendations.

Mastromarco & Simar (2018) study the problem of reliability of the parametric analysis of Global Frontiers. According to the authors, there is an opinion that parametric models make a much better analysis of the production process in terms of elasticity. However, they note, this is true if the chosen parametric model is adequate to the true frontier. They agree with the view of Florens and Simar (2005) that most of these models have defects caused by the heterogeneity of distribution and distant data points. It occurs in case the selected parametric model is a reasonable approximation of the true frontier. But then, Florens and Simar (2005) consider the majority of estimation methods of the stated parametric frontier models experience certain downsides supposing there is the heterogeneity of the efficiency distribution over the input values and/or if data points are peripheral.

Pettis (2020) focused on the problem of possible inaccuracy in statistics data, particularly in China. The scientist explains this issue with the specific practice. Particularly, unproductive investments are not written off, and it causes an overestimation of GDP.

Madsen (2014) believes that it is advisable to choose a priori a large developed country as a standard of efficiency. When studying the role of the education level, he defines the WTF as the maximum cumulative factor productivity of the United States and the United Kingdom, provided that small countries (such as Norway) cannot influence it.

The world's technological frontiers are built based on the authors' theoretical models in several works. Thus, Acemoglu et al. (2017) built a model of technologically interconnected countries. In this model, greater inequality between successful and unsuccessful entrepreneurs contributes to the country's achievement of the global technological frontier. Countries with "cutthroat" capitalism are becoming technology leaders, and the rest of the world is taking advantage of their achievements.

Qin Ye et al. (2021) proposed a model showing the positive impact of reducing the technology gap on the environment and confirmed it based on Chinese statistics.

The work of Caselli & Coleman II (2006) can be considered the closest to the topic of the proposed study. This study examines the differences between countries in the aggregate production function, where skilled and unskilled labour are imperfect substitutes. The authors build a WTF based on the neoclassical theory, according to which firms maximize their profits with a given production function. The authors analyze various forms of the production function, where the arguments are the stock of labour, as well as the amount of skilled and unskilled labour $L_s$, $L_u$. To determine the efficiency coefficients of these types of labour $A_s$ and $A_u$ a nonlinear constraint is introduced. The authors define the parameters of their model by regression and then build the WTF in a logarithmic coordinate system as an envelopment of the corresponding national frontiers.

Vandenbussche et al. model (2006) present the world as consisting of a finite number of economies with a single population that does not trade internationally. Just like the model of Caselli–Coleman the model proposed by the authors is based on the neoclassical theory of firm behaviour. A special feature of their model is the image of firms as local monopolists. In this model, each time at the beginning of the next period, the firm chooses a certain combination: 1) imitation activities aimed at introducing the world frontier technologies and 2) innovations at the local technological frontier. The authors concluded via mathematical analysis of their model that the impact of the stock of qualified human capital on economic growth is getting stronger when the economy is closer to the technological frontier. Based on data covering 19 OECD countries between 1960–2000, the authors conducted an empirical analysis of the model and found out why previous studies failed to find a positive relationship between the schooling level and subsequent increase in well-to-do countries.

To test basic hypotheses how exactly human capital helps nations accelerate economic growth, Islam (2010) decided, in his words, to adhere to the same empirical methodology as Vandenbussche et al. Through the example of 87 countries of different levels of development, he deduced that the effect of a skilled human capital increases while approaching the technological frontier for high- and middle-income countries, while young workers with the secondary education accelerate economic growth in low-income countries.

Studying the Caselli – Coleman model, Krüger (2017) notes that even though the main result of their study is robust to using the nonparametric approach; the remaining results are sensitive to alternative definitions of the skilled and unskilled labour, data sources, and measurement approach options.

Caselli & Coleman's idea of incomplete interchangeability of skilled and unskilled labour is also developed by Growiec (2008, 2012). However, unlike its predecessors, he applies a nonparametric approach. The author draws attention to the significant internal heterogeneity of the United States, due to its large size. According to his calculations, US data disaggregated to the level of individual states, first, show that the production can be more efficient than the US average. Secondly, this use of national American data together with the data from individual states enables, in his opinion, to build the WTF line more accurately.

Güvercin (2020) argued that developing countries should implement rigid labour market policies to be closer to the global technological frontier.

Braun et al. (2021) and Alheet et al., (2021) showed that country can fall into the middle-income trap when moving to the world technological frontier if its economic policy and financial market do not improve accordingly.

Kumar & Singh (2019) studied the human barriers that hinder the spread of technological innovation. Researchers showed the negative impact of the genetic distance of the country's populations to the world's technology frontier, using the country-specific economic complexity index (ECI).

Keller (2001) assessed the importance of the geographical factor of technology dissemination, taking into account international trade, foreign direct investment and communication channels between 1970 – 1995.

A similar analysis of the international trade impact is developed by Melitz & Redding (2021) and Siemiątkowski (2017).

Stakanov (2020) indicates that the consequence of the digitization and robotization processes is the spread of industrialization 4.0, which is manifested in the accelerated robotization of the world economy and accelerated data exchange in global production processes.

Shubin Yang et al. (2021) argued that non-exporters are the fastest to approach the technological frontier, followed by non-permanent exporters, followed by regular exporters. The researchers explain this issue that more well-known exporters are already closer to the technological frontier.

Ly Dai & Thuy Hoan (2018) used the model of open multi-country overlapping generations economy to analyze the directions of net capital flows. Their study is based on 175 countries' data. They found that approaching the global technological frontier reduces net total capital inflows.

In some works, the dynamics of the WTF was chosen as the subject of research. Thus, Cantner & Hanusch (1999) consistently build annual WTF between1960 – 1990 period based on data for 87 countries. The applied coordinate system « $L/Y - K/Y$ », presented WTF in the form of unit isoquants, which is the most informative form.

Lafuente et al. (2020) use the WTF concept to analyze two types of improving national efficiency: 1) borrowing existing technologies or reallocating resources into more efficient activities (entrepreneurship according to Kirzner) and 2) introducing fundamentally new technologies based on generating new knowledge (entrepreneurship according to Schumpeter). In the first case, the outsider country follows the new WTF without crossing it. At the same time, if it has a relatively high stock availability, the level of its labour productivity may temporarily be even higher than that of the leader. However, in the long run, the Schumpeterian type of entrepreneurship shifts the WTF so significantly that even in conditions of lower capital-labour ratio, the leading country begins to outstrip the outsider in terms of labour productivity.

Krüger (2020) analysed the dynamics of the total factor productivity during the 1970-2014 years for 93 countries. The researcher's results are based on the evaluation of the Malmquist index. This study revealed a dynamic shift in the position of certain groups of countries on the global technological frontier and the frontier itself.

Mastromarco & Simar (2021) argued that human capital impacts the country's movement towards the global technological frontier, but doesn't impact its shift.

## 3. METHODOLOGY

### 3.1. Balance of technological and economic efficiency indicators

Let us consider the economy of a particular country X, a few assumptions about which will be made. First, let us assume that only two factors of production are used at the macro level, each of which is paid at its rate. Second, let us assume that the material costs, taxes, and economic profits of production firms[1] equal zero. Then the nominal income of production firms must be equal to the sum of the nominal income of the owners of the production factors:

$$PY = RK + WL \quad (1)$$

where $P$ – a unit price of the created product; $Y$ – physical volume product produced in the country per unit of time; $K$ and $L$ – physical values of fixed capital and the labour force; $R$ and $W$ – nominal rates of rent and wage pay.

The resulting equation of the nominal income balance can be represented in two forms. The first form is a function of distributing the product among the owners of factors of production:

$$Y = \frac{R}{P}K + \frac{W}{P}L \quad \Rightarrow \quad Y = rK + wL \quad (2)(3)$$

where $r = R/P$, $w = W/P$ – real rental price of capital, from the point of view of firms, real hiring price of labour. We will note that the real prices of factors of production for their owners are not identical with the real prices from the point of view of production firms. They are measured in the product they sell for firms, and for factor owners, they are measured in the products they buy. Real, from the point of view of owners, prices of their factors show how many units of a product they can buy in one «resource-hour» sold. Real price factors from a firm's point of view show how many units of product firms must sell to buy one «resource-hour». Inverse values are indicators of economic efficiency. They show how many «capital-hours» and «man-hours» firms can buy by selling a single unit of production:

$$\frac{P}{R} = \frac{1}{r} \qquad \frac{P}{W} = \frac{1}{w} \quad (4)(5)$$

The second equivalent form of nominal income balance is a function of product price distribution:

$$P = \frac{K}{Y}R + \frac{L}{Y}W \quad \Rightarrow \quad P = kR + lW \quad (6)\ (7)$$

where $k = K/Y$, $l = L/Y$ – capital intensity and labour intensity. Inverse values are indicators of technological efficiency:

$$\frac{Y}{K} = \frac{1}{k} \qquad \frac{Y}{L} = \frac{1}{l} \quad (8)\ (9)$$

They show how many units of product firms can produce using one «resource-hour».

[1]The term neoclassical theory of «production firms» according to the version used by N. Gregory Mankiv, 1992.

Both forms of income balance are reduced to the following expression:

$$rk + wl = 1 \quad (10)$$

it will be designated as the balance of technological and economic efficiency indicators. It can be viewed from two points of view. Firstly $k$ and $l$ can be considered the coordinates of the current technological state of a country, and $r$ and $w$ are parameters of the volume distribution function of its product. Secondly, $r$ and $w$ can be considered the coordinates of the current economic state of a country, and $k$ and $l$ are the parameters of the price distribution function of its product. It is the balance nature of this equation that creates a conflict between two goals – improving technological and economic efficiency: reducing $k$ then $l$ leads to the cost rising $r$ and $w$ increase in the cost of factors for production firms, and vice versa.

### 3.2. Geometry of elementary efficiency frontiers

Let us consider two countries – $X_1$ and $X_2$, characterized by their states – $T_1(l_1, k_1)$, $E_1(w_1, r_1)$ and $T_2(l_2, k_2)$, $E_2(w_2, r_2)$ none of which is absolutely the best. Thus, both countries are sheer leaders, either of which has the highest level of one of the efficiency indicators:

$$l_1 < l_2 \quad k_1 > k_2 \quad w_1 > w_2 \quad r_1 < r_2 \quad (11)$$

The efficiency frontiers constructed using the actual state.[2] The envelopment method has the form shown in Figure 1.

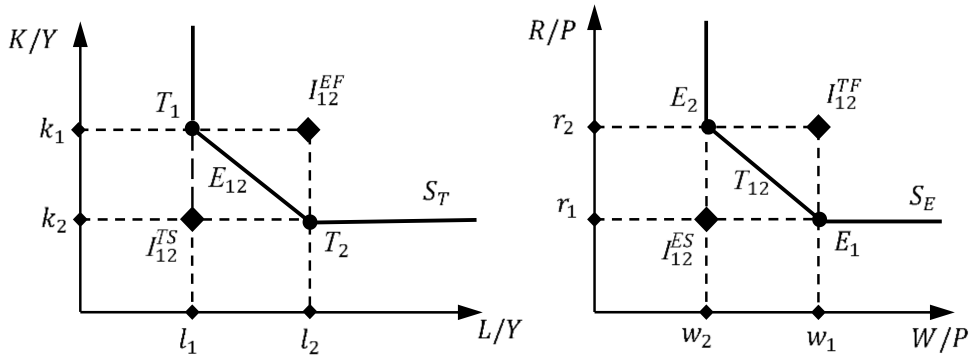


**Figure 1. International efficiency frontiers by the state envelopment method**
*Source:* Authors' model

The left part of Figure 1 shows the international technological frontier and the right part shows the international economic frontier. Point $I_{12}^{TS}(l_1, k_2)$, represents an ideal technological state when each of the factors of production is used with the highest efficiency for the two countries. Point $I_{12}^{ES}(w_2, r_1)$ is an ideal economic state when the factors of production are the cheapest.

Geometrically, the national balance of efficiency indicators is a straight line intersecting the coordinate axes of the technological plane at points $l = 1/w_X$, $k = 1/r_X$; and the economic plane – at points $w = 1/l_X$, $r = 1/k_X$. Due to this property, real states of the countries and real distribution functions can be represented on opposite planes as straight lines, virtual functions and virtual states – as their intersection

[2] In the study of dynamical systems in mathematics and physics, the concept of phase state is used. By this analogy, the proposed model uses the concepts of technological state, which is characterized by the productivity of production factors, and economic state, which is characterized by real prices of these factors. Envelopments are built for both data sets, which will be called state envelopments.

points, and points of ideal states and ideal functions – as straight lines intersecting the coordinate axes. A geometric interpretation of the states and functions presentation is shown in Figure 2.

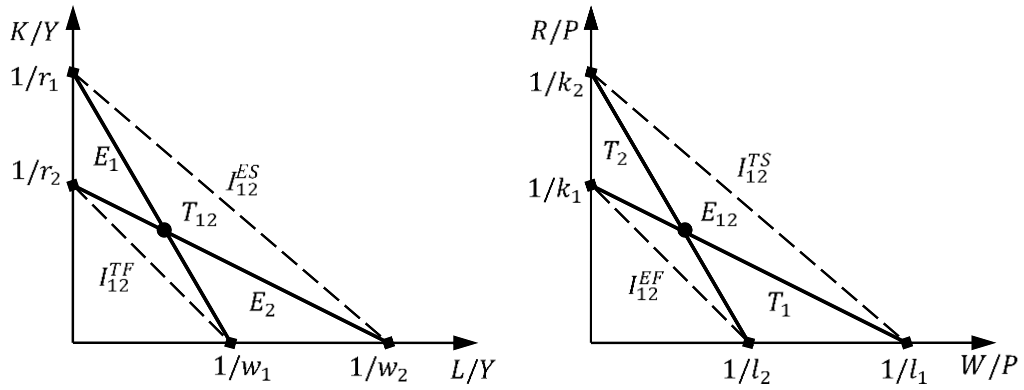


**Figure 2. Presentation of real states and real distribution functions in the form of lines**
*Source:* Authors' model

On the left of Figure 2 straight lines, $E_1$ and $E_2$ are real economic states of countries (or real functions of distribution of their product). On the right of Figure 2, $T_1$ and $T_2$ are real technological states (or real price distribution functions). Straight-line $I_{12}^{EF}$ is an ideal function of the price distribution. In the previous figure, it is represented by a dot $I_{12}^{EF}(l_2, k_1)$ on the technological plane, in which the parameters of this function are the best (the intensity of using factors of production is the highest). Straight-line $I_{12}^{TF}$ is an ideal function of the price distribution. In the previous figure, it is represented by a dot $I_{12}^{TF}(w_1, r_2)$ on the technological plane, where the parameters of this function are the best (the intensity of using factors of production is the highest).

Thus, it is convenient to construct efficient frontiers using the function envelopment method as projections on opposite planes, where they will be an envelopment of points. The overall picture looks like this. The international technological frontier, built on the technological plane using the state envelopment method ($S_T$), is convex to the bottom: its upper extreme section is vertical, and its lower extreme section is horizontal. The best technological states are located at its corner points.

The international economic frontier, built on the same plane using the envelopment method of price distribution functions ($F_E$), is convex upwards: its upper extreme section is horizontal, and its lower extreme section is vertical. The worst technological states are located at the corner points of this frontier.

A similar geometric situation is on the economic plane. The international economic frontier, constructed using the method of countries' states envelopment ($S_E$), is convex to the bottom. The best economic states are angular on it. The international technological frontier built via the method of national distribution functions envelopment ($F_T$), is convex upwards. The worst economic states are angular on it.

Thus, on both planes, the points of all countries are located between the technological and economic frontiers. Virtual countries form with these frontiers on the technological plane the intersection points of the beam of the outsider country's capital-labour ratio $X$ ($\kappa_X = k_X/l_X$): $\Sigma_{XT}(l_{\Sigma X}, k_{\Sigma X})$ – at the international technological frontier on the method of countries' states envelopment and $\Phi_{XE}(l_{\Phi X}, k_{\Phi X})$ – at the international economic frontier using the method of national distribution functions envelopment. On the economic plane, the intersection points of the beam of the relative capital price of the same outsider country $X$ ($\rho_X = r_X/w_X$) with both frontiers form two other virtual countries: $\Sigma_{XE}(w_{\Sigma X}, r_{\Sigma X})$ – at the international economic turn by the method of states envelopment and $\Phi_{XT}(w_{\Phi X}, r_{\Phi X})$ – at the international technological frontier using the function envelopment method.

The ratio of coordinates of a real country *X* and its virtual countries represent the distances to the corresponding frontiers:

$$\mathrm{d}(\Sigma_{\mathrm{XT}}) = \frac{l_{\Sigma X}}{l_X} = \frac{k_{\Sigma X}}{k_X} \leq 1 \qquad \mathrm{d}(\Phi_{\mathrm{XE}}) = \frac{l_X}{l_{\Phi X}} = \frac{k_X}{k_{\Phi X}} \leq 1 \qquad (12)\ (13)$$

$$\mathrm{d}(\Sigma_{\mathrm{XE}}) = \frac{w_{\Sigma X}}{w_X} = \frac{r_{\Sigma X}}{r_X} \leq 1 \qquad \mathrm{d}(\Phi_{\mathrm{XT}}) = \frac{w_X}{w_{\Phi X}} = \frac{r_X}{r_{\Phi X}} \leq 1 \qquad (14)\ (15)$$

$$\mathrm{d}(\Sigma_{\mathrm{T}}\Phi_{\mathrm{E}})_{\mathrm{X}} = \frac{l_{\Sigma X}}{l_{\Phi X}} = \frac{k_{\Sigma \mathrm{X}}}{k_{\Phi X}} < 1 \qquad \mathrm{d}(\Sigma_{\mathrm{E}}\Phi_{\mathrm{T}})_{\mathrm{X}} = \frac{w_{\Sigma X}}{w_{\Phi X}} = \frac{r_{\Sigma X}}{r_{\Phi X}} < 1 \qquad (16)\ (17)$$

Inverse values $\mathrm{d}(\Sigma_{\mathrm{XT}})^{-1}$ and $\mathrm{d}(\Sigma_{\mathrm{XE}})^{-1}$ show how many times the volume of production and the price level must be raised; inverse values $\mathrm{d}(\Phi_{\mathrm{XE}})^{-1}$ and $\mathrm{d}(\Phi_{\mathrm{XT}})^{-1}$ indicate the number of times to reduce them. Values $\mathrm{d}(\Sigma_T\Phi_E)_{\mathrm{X}}$ and $\mathrm{d}(\Sigma_{\mathrm{E}}\Phi_{\mathrm{T}})_{\mathrm{X}}$ can be interpreted as distances between opposite frontiers in the direction of country *X*.

The geometric interpretation of all four frontiers and virtual countries on them is shown in Figure 3.

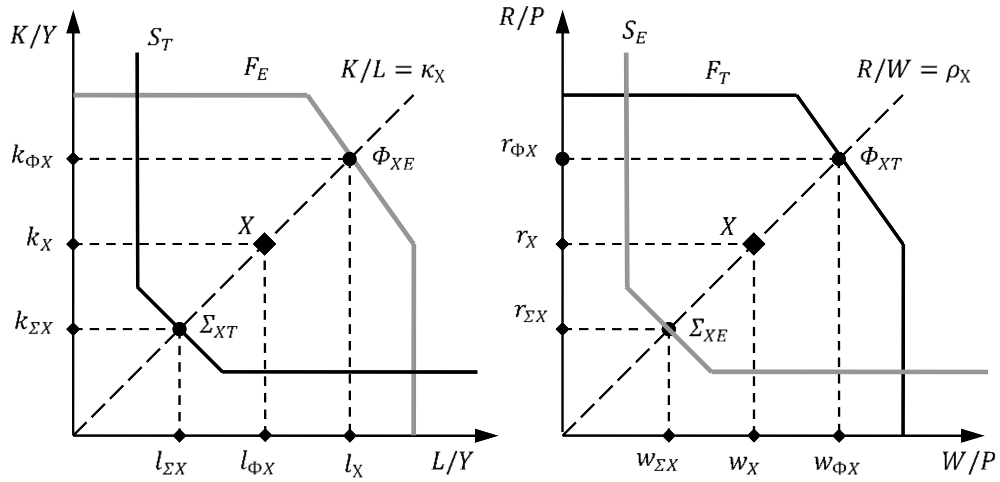


**Figure 3. International frontiers of efficiency and virtual countries on them**
*Source: Authors' model*

Figure 3 shows international technological frontiers in black bold lines and international economic frontiers in gray bold lines. $S_T$, $S_E$ are international frontiers built using the state envelopment method; $F_E$, $F_T$ – using the function envelopment method.

Therefore, frontiers $S_T$ and $F_E$ are mutually opposite envelopments in the coordinate system «$L/Y - K/Y$», and frontiers $S_E$ and $F_T$ are mutually opposite in the coordinate system «$W/P - R/P$».

### 3.3. International frontiers of efficiency as a solution to the linear programming problem

From an algebraic point of view, the DEA method is a linear programming problem. Regarding this study, this method results in two input-oriented tasks that, together with the nominal income balance, make up the first part of the proposed model:

$$\begin{cases} \min \tau_j \qquad \forall j \\ \sum_{i=1}^{n} \lambda_i Y_i \geq Y_j \quad \sum_{i=1}^{n} \lambda_i K_i \leq \tau_j K_j \quad \sum_{i=1}^{n} \lambda_i L_i \leq \tau_j L_j \\ \lambda_i > 0 \end{cases} \tag{18}$$

$$\begin{cases} \min \varepsilon_j \qquad \forall j \\ \sum_{i=1}^{n} \lambda_i P_i \geq P_j \quad \sum_{i=1}^{n} \lambda_i R_i \leq \varepsilon_j R_j \quad \sum_{i=1}^{n} \lambda_i W_i \leq \varepsilon_j W_j \\ \lambda_i > 0 \end{cases} \tag{19}$$

$$P_j Y_j = R_j K_j + W_j L_j \tag{20}$$

where $j$ is a country number, $i = 1, \ldots n$ are numbers of countries with which it is compared; $\min\tau_j$, $\min\varepsilon_j$ are indicators of general technological and, accordingly, the overall economic efficiency of the country, the value of which is not more than one.

Value $1/\min\tau_j$ shows the degree of changes required to reach the technological efficiency frontier: how many times country $X_j$ must reduce the volume of factors of production with the same volume of product. Similarly, the value of $1/\min\varepsilon_j$ shows the degree of changes required to reach the frontier of economic efficiency: how many times country $X_j$ must reduce the prices of factors of production at the same level of product prices.

Frontiers $S_T$ and $S_E$ represent the envelopment of the best technological and, accordingly, economic states. Replacing minimization tasks $\tau_j$, $\varepsilon_j$ by input-oriented tasks of their maximizing results in the construction of two more frontiers of efficiency – $F_T$ and $F_E$, which are the envelopments of the best distribution functions – according to the product and its price:

$$\begin{cases} \max \varepsilon_j \qquad \forall j \\ \sum_{i=1}^{n} \lambda_i P_i \leq P_j \quad \sum_{i=1}^{n} \lambda_i R_i \geq \varepsilon_j R_j \quad \sum_{i=1}^{n} \lambda_i W_i \geq \varepsilon_j W_j \\ \lambda_i > 0 \end{cases} \tag{21}$$

$$\begin{cases} \max \tau_j \qquad \forall j \\ \sum_{i=1}^{n} \lambda_i Y_i \leq Y_j \quad \sum_{i=1}^{n} \lambda_i K_i \geq \tau_j K_j \quad \sum_{i=1}^{n} \lambda_i L_i \geq \tau_j L_j \\ \lambda_i > 0 \end{cases} \tag{22}$$

where $\max\tau_j$, $\max\varepsilon_j$ are indicators of the overall technological and, accordingly, general economic inefficiency of the country, the value of which is not less than one.

### 3.4. Factorization of gross values

Let us consider the production method for calculating the gross domestic product of a certain country, making several assumptions about it.

Let us assume that net product taxes $NPT$ are directly proportional to the gross value added $GVA$:

$$NPT = t \cdot GVA \qquad t > 0 \tag{23}$$

where $t$ is the rate of the net product taxes. Then the gross domestic product $GDP$ equals to

$$GDP = (1 + t) \cdot GVA \tag{24}$$

Let us assume further that the intermediate consumption is directly proportional to the gross output $GO$ :

$$IC = a \cdot GO \qquad 0 < a < 1 \tag{25}$$

where $a$ is a direct material cost rate. Let's present all three gross values $GO$, $GDP$ and $GVA$ as products of a physical volume of a particular output and a monetary value of its unit:

$$GO = P \cdot X \qquad GDP = P \cdot Q \qquad GVA = V \cdot Q \qquad (26)(27)(28)$$

where $P$, $V$ are a market price and average added value, and $X$, $Q$ is total and net output. In terms of its content, the average value-added $V$ is a macroeconomic analogue of the microeconomic concept of average total costs $ATC$ firms.

As a result, the following equations of gross domestic product and gross output are obtained:

$$PQ = VQ + tVQ \qquad PX = VQ + aPX \qquad (29)(30)$$

Based on Leontief «input-output» model, value $(1 - a)X$ is considered as a final output $Y$.

Then we get a system of four equations forming a proposed model:

$$\begin{cases} X = aX + Y & (31) \\ Q = Y + tY & (32) \\ P = V + tV & (33) \\ VQ = PY & (34) \end{cases}$$

In this system, the first three equations are distribution equations. The fourth equation shows the equivalence of two forms of representation of gross value added and means that in monetary terms, the net output is equal to the final output. The number of the unknowns in this system $X$, $Y$, $Q$, $P$, $V$ is more than the number of equations. To solve it, it is necessary to determine any of these unknowns based on a different model. The only value of this kind is $P$ since all aggregated statistics data are expressed in prices. This problem is solved further.

The proposed model of the national production account compares the theoretical values of the total, net and final output. As it follows from its equations, the total output is more than the final one, and the final one is less than the net one. The net output is less than the total one only if the product of the coefficients $(1 - \text{a})$ and $(1 + t)$ is less than one:

$$Q = (1 - a)(1 + t)X \qquad \Rightarrow \qquad Q < X \Leftrightarrow (1 - a)(1 + t) < 1 \tag{35}$$

In monetary terms, this means that inequality must be done

$$VQ + tVQ < PX \tag{36}$$

Since in practice $GDP < GO$, then this means that the following inequality holds for the three outputs:

$$X > Q > Y \tag{37}$$

In the national production account, it is displayed as follows:

$$GO - GDP = IC - NPT > 0 \quad (38)$$

### 3.5. Selecting the form of the efficiency indicator balance

Let us consider the multipliers making up two theoretical forms of the gross value added, from the point of view of the goals of two macroeconomic entities – the state and private production firms.

All other things being equal, an increase in the tax rate $t$ increases the price level

$$P = (1 + t)V \quad (39)$$

In this way, the state can use more factors of production, for example, in the public sector. It can be qualified as an increase in socio-economic efficiency. On the other hand, raising the tax rate increases net output,

$$Q = (1 + t)Y \quad (40)$$

which authorizes the state to provide more social services. It can be qualified as an increase in «socio-technological» efficiency.

Thus, the state's goal is to increase the price level $P$ and net output $Q$. It should be noted that the fight against inflation, which is considered one of the priorities of the state's economic policy, actually does not implement its own goal as a macroeconomic entity, but the goal of owners of factors of production and money balances. Real incomes decrease for the former, first of all, employees as a result of inflation, for the latter – real cash balances. If the control over the actions of the State on their part is weakened, then the goal of raising the tax rate and, as a result, the price level is implemented more and more. This phenomenon was most common in the pre-industrial era, and in modern conditions, it is spread in countries with authoritarian and totalitarian regimes.

As for the goal of private production firms, it is the opposite of state one. They are interested in increasing the average value-added $V$ and the final output $Y$ without taxes. For them an increase in the tax rate $t$, all other things being equal, leads to a decrease in the average value-added and final output:

$$V = \frac{P}{1 + t} \qquad Y = \frac{Q}{1 + t} \quad (41)(42)$$

As a result of the decline in the average value-added, they will have to use fewer factors of production, which will become more expensive for them. Then it follows that the decrease in the final output reduces the physical volume of products produced, and accordingly, their real income. Thus, decrease $V$ can be qualified as a drop-in private economic efficiency, while reduction $Y$ can be understood as a drop-in private technological efficiency.

Two theoretical forms of expressing the gross value added lead to two different forms of the balance of efficiency indicators:

$$PY = RK + WL \quad \Rightarrow \quad \frac{R}{P} \cdot \frac{K}{Y} + \frac{W}{P} \cdot \frac{L}{Y} = 1 \quad (43)$$

$$VQ = RK + WL \quad \Rightarrow \quad \frac{R}{V} \cdot \frac{K}{Q} + \frac{W}{V} \cdot \frac{L}{Q} = 1 \quad (44)$$

According to the first of these forms, an increase in the tax rate $t$ will lead to an increase in socio-economic efficiency and a decrease in private technological efficiency. According to the second form, an increase in the tax rate will increase socio-technological efficiency and reduce a private economic one.

Let us note that the equations of the proposed model do not include taxes that constitute a part of the value-added because with their help the State does not create a new part of the output, but withdraws a part of the already created one.

When choosing between these two methods of decomposition of the gross value added, we will take into account the one helping to consider more factors. There are two such factors in the proposed model that is the rate of direct material costs $a$ and the rate of net product taxes $t$. Decomposing the gross value added into final output $Y$, it is possible to account for the rate of direct material costs: $Y = (1 - a)X$. However, when decomposing gross value-added into net output, the rate of net product taxes can be included: $Q = (1 + t)(1 - a)\,X$.

On the other hand, in both factorization methods the first multipliers $P$ and $V$ are not directly observed. However, in national accounts, all values are calculated in prices, not in the average value-added. Thus, $P$ is no longer decomposed, while the average value-added can be represented as $V = P/(1 + t)$. Based on these considerations, the proposed study uses the decomposition of the gross value added into its average value $V$ and a net output $Q$:

$$GVA = V \cdot Q = [P/(1 + t)] \cdot [(1 + t)(1 - a)X] \qquad (45)$$

Values $a$ and $t$ are determined directly from the data of national accounts. Value $X$ is determined from the given equation. Value $P$ must be found independently.

### 3.6. Factorization of export-import flows

In modern conditions, the most well known and reasonable method of comparative analysis of national economies is the purchasing power parity conversion coefficient factor. It is calculated according to the International Comparison Program supervised by the World Bank. The World Bank recommends using this indicator for spatial comparison of GDP and price levels, grouping countries by their per capita volume indices and other purposes. However, the World Bank does not recommend the use of PPP as a «precise measure to establish strict rankings of countries» and warns that PPP estimates are not a part of the national official statistics.[3] The purpose of this study, though, is not just to establish such a rating, but also to determine the distances between leaders and outsiders, thus, these official statistics was chosen as the only source of the data used. In addition, according to the PPP method, the countries under the study are considered in isolation from each other, without taking into account their mutual influence through international trade.

Based on these considerations, the proposed study attempted to construct an alternative indicator of relative prices to compare different countries.

The proposed model for determining the relative price of products of a given country is based on the fact that $-export value of a single country to the rest of the world, does not coincide with the $-import value with the rest of the world from this country. Similarly, the values of a country's imports from the World and World exports to that country do not coincide. Since the physical volume of products must remain unchanged when crossing the country's border, it is logical to assume that this difference occurs due to price discrepancies in the domestic and foreign markets.

Proceeding from these considerations, we will study the domestic market of country $C$ where foreign and domestic export-import firms operate. Then the cash flows caused by export-import operations can be decomposed into multipliers as follows:

- imports by the World from Country $IWC$ – as a product of the price of export products on the domestic market $EP_{DM}$ by the physical volume of their import by the World $IZ_W$;

[3] PPP conversion factor, GDP (LCU per international $). Details. – https://data.worldbank.org/indicator/PA.NUS.PPP

- exports by the World to Country $EWC$ – as a product of the price of imported products on the domestic market $IP_{DM}$ by the physical volume of their exports by the World $EZ_W$;
- imports by Country from World $ICW$– as a product of the price of imported products on the domestic market $IP_{DM}$ by the physical volume of their imports by Country $IZ_C$;
- exports by Country to World $ECW$– as a product of the price of export products on the domestic market $EP_{DM}$ by the physical volume of their exports by the Country $EZ_C$.

Each of these cash flows can be considered as a value, demand $DV$ or supply $SV$ by the domestic and foreign exporters and importers:

$$IWC = EP_{DM} \cdot IZ_W = DV_{FI} \qquad EWC = IP_{DM} \cdot EZ_W = SV_{FE} \tag{46)(47}$$

$$ICW = IP_{DM} \cdot IZ_C = SV_{DI} \qquad ECW = EP_{DM} \cdot EZ_C = DV_{DE} \tag{48)(49}$$

Similarly, the same cash flows on the foreign trade market of country $C$ can be represented as:

$$EWC = IP_{FM} \cdot EZ_W = DV_{FE} \quad IWC = EP_{FM} \cdot IZ_W = SV_{FI} \tag{50)(51}$$

$$ECW = EP_{FM} \cdot EZ_C = SV_{DE} \quad ICW = IP_{FM} \cdot IZ_C = DV_{DI} \tag{52)(53}$$

where $IP_{FM}$, $EP_{FM}$ are import and export prices (for country $C$) products on its foreign trade market.

When crossing the country's border, the physical volume of the same commodity flow must remain the same, therefore:

$$EZ_W \equiv IZ_C \qquad EZ_C \equiv IZ_W \tag{54)(55}$$

However, its value will change due to the changes in the price:

$$IP_{DM} \neq IP_{FM} \qquad \Rightarrow \qquad EWC \neq ICW \tag{56}$$

$$EP_{DM} \neq EP_{FM} \qquad \Rightarrow \qquad ECW \neq IWC \tag{57}$$

which is observed in real conditions. Taking this into account, we will make the following ratios of demand values: $DV_{DE}$, $DV_{DI}$, $DV_{FE}$, $DV_{FI}$ and supply $SV_{DE}$, $SV_{DI}$, $SV_{FE}$, $SV_{FI}$, so that the resulting values can be interpreted as relative prices – the relative price of demand for domestic products from foreign importers $RDP_{FI}$, the relative price of demand for domestic products from domestic exporters $RDP_{DE}$, the relative supply price of foreign products by foreign exporters $RSP_{FE}$, the relative supply price of foreign products by domestic importers $RSP_{DI}$:

$$RDP_{FI} = \frac{DV_{FI}}{SV_{DE}} = \frac{EP_{DM} \cdot IZ_W}{EP_{FM} \cdot EZ_C} = \frac{IWC}{ECW} \tag{58}$$

$$RDP_{DE} = \frac{DV_{DE}}{SV_{FI}} = \frac{EP_{DM} \cdot EZ_C}{EP_{FM} \cdot IZ_W} = \frac{ECW}{IWC} \tag{59}$$

$$RSP_{FE} = \frac{SV_{FE}}{DV_{DI}} = \frac{IP_{DM} \cdot EZ_W}{IP_{FM} \cdot IZ_C} = \frac{EWC}{ICW} \tag{60}$$

$$RSP_{DI} = \frac{SV_{DI}}{DV_{FE}} = \frac{IP_{DM} \cdot IZ_C}{IP_{FM} \cdot EZ_W} = \frac{ICW}{EWC} \tag{61}$$

The inverse values will represent the relative supply prices of domestic products $RSP_{DE}$, $RSP_{FI}$ and relative demand prices for foreign products $RDP_{DI}$, $RDP_{FE}$.

It is logical to assume that a country is more successful, the greater the demand for its products. On this basis, all prices for foreign products and supply prices for domestic products cannot be considered as a basis for determining the effectiveness of a country. Comparing the two demand prices for domestic products in the intra-trade market, preference should be given to $RDP_{FI}$ – the relative price of demand for domestic products from foreigners. Hence, it is this value that we will use as a relative price of products of this country:

$$P = RDP_{FI} = \frac{IWC}{ECW} \tag{62}$$

As is customary in macroeconomic theory, we can estimate the physical volumes of products and capital by dividing their nominal market values by this relative price. To do this, let us assume that these values are products of $EP_{DM}$ – prices of export products on the intra-trade market and corresponding physical volumes. Then, as a result of dividing the nominal market values by $RDP_{FI}$, we get an estimate of physical volumes in $EP_{FM}$ – export prices on the foreign trade market.

### 3.7. Determination of physical volumes of factors of production and their nominal prices

We will solve this problem in several stages. The physical volume of labour $L$ will be defined based on the concept of human capital:

$$L = HCI \cdot TLF \tag{63}$$

where $HCI$ is Human Capital Index, $TLF$ is a total labour force of a country. According to this approach, unemployment is a manifestation of inefficient use of the labour force, and that is why it is taken into account in the amount of labour $L$. Similarly, in the proposed model, unused fixed capital is not excluded from the physical quantity $K$.

Let us further assume that the market value of productive capital $MVPC$ is equal to the gross fixed assets except for the cost of dwellings:

$$P \cdot K = MVPC = GFA - D \tag{64}$$

Hence, the physical volume of productive capital will be equal to:

$$K = \frac{MVPC}{P} \tag{65}$$

Now, based on the production account and generation of income account, let us define the allocation of the gross value added on factor income. Let us assume that the gross value added is divided by the factor income in proportion to the ratio of gross operating surplus $GOS$ to the compensation of employees $CE$:

$$GVA = GDP - NPT = R \cdot K + W \cdot L \tag{66}$$

$$\frac{R \cdot K}{W \cdot L} = \frac{GOS}{CE} \tag{67}$$

Then the total factor income, taking into account net taxes, will be determined as follows:

$$R \cdot K = TCI = GVA \cdot \frac{GOS}{GOS + CE} \tag{68}$$

$$W \cdot L = TLI = GVA \cdot \frac{CE}{GOS + CE} \tag{69}$$

where $TCI$, $TLI$ is total capital income and labour income respectively. Since the volumes of labour and capital are already known, the nominal wage rate and nominal rent rate, are:

$$W = \frac{TLI}{L} \qquad R = \frac{TCI}{K} \qquad (70)(71)$$

Based on the obtained absolute values $V$, $Q$, $R$, $K$, $W$, $L$ relative values can be calculated as follows:

$$k = K/Q, \qquad l = L/Q, \qquad r = R/V, \qquad w = W/V \qquad (72)(73)\ (74)(75)$$

necessary to build efficiency frontiers following the decomposition of gross value added into the average value and net output: $GVA = V \cdot Q$.

## 4. EMPIRICAL RESULTS AND DISCUSSION

### 4.1. Initial data and calculated efficiency indicators

Traditional problems hindering macroeconomic research are inconsistency and incompleteness, and sometimes dubiously of the initial data. With this in mind, the proposed study refers to the data taken exclusively from the official websites of the leading international organizations and the US Federal Reserve System. Therefore, national accounts indicators, the gross domestic product, taxes less subsidies on products, compensation of employees, gross operating surplus were obtained from the website of the «UNdata»; the gross fixed assets and dwellings were obtained from the website of the OECD Statistics and Data Directorate «OECD.Stat»; the data on export and import flows of goods – from the WITS of World Bank; data on Human Capital Index and the total labour force – from the World Bank Open Data website. Because on the UN and OECD websites the values are expressed in units of the national currency (NC), a conditional exchange rate was applied. This rate was calculated as the ratio of nominal GDP expressed in the national currency of a given country to a similar value expressed in current US dollars:

$$ER = \frac{GDP^{NC}}{GDP^{\$}} \tag{76}$$

The necessary data were obtained from the Federal Reserve Economic Data.

The available statistics imposed significant restrictions on the choice of the study period and the list of countries studied. At the start of the research, 2017 was the last year the OECD provided data on fixed assets and dwellings only for 13 countries – Austria, Belgium, the Czech Republic, Denmark, Finland, France, Germany, Israel, Italy, Luxembourg, the Netherlands, Slovenia and the United Kingdom. Thus, the proposed model was built for this group of countries based on data for 2017.

The initial statistical data, the results of calculating conditional exchange rates and relative prices are shown in Tables 1 and 2, and the calculated theoretical efficiency indicators are shown in Table 3.

Table 1

National accounts indicators at current prices, million units of national currency / current U.S. Dollars

| Country | Gross domestic product | Taxes fewer subsidies on products | Compensation of employees | Gross operating surplus | Gross domestic product | National currency to U.S. dollar exchange rate |
|---|---|---|---|---|---|---|
| | *NC* | *NC* | *NC* | *NC* | *USD* | $GDP^{NC}/GDP^{\$}$ |
| | $GDP^{NC}$ | $NPT^{NC}$ | $CE^{NC}$ | $GOS^{NC}$ | $GDP^{\$}$ | $ER$ |
| Austria | 370295,8 | 39962,8 | 176072,6 | 121101,9 | 416207,3 | 0,889690786 |
| Belgium | 446364,9 | 48230,6 | 219279,1 | 151452,4 | 501522,9 | 0,89001898 |
| Czechia | 5047267 | 518123 | 2089199 | 1965209 | 218628,9 | 23,08600098 |
| Denmark | 2175106 | 285468 | 1118290 | 666465 | 332121,1 | 6,54913524 |
| Finland | 225836 | 30594 | 104558 | 81779 | 255016,5 | 0,885574071 |
| France | 2295063 | 251066 | 1198018 | 664269 | 2588740,9 | 0,8865557 |
| Germany | 3244990 | 322662 | 1693037 | 980040 | 3673506,3 | 0,883349513 |
| Israel | 1271555,3 | 124136,9 | 574463,1 | 530496,5 | 352667,9 | 3,60553172 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Italy | 1736592,9 | 178797 | 684054,1 | 600904,5 | 1956950,5 | 0,887397458 |
| Luxemburg | 56814,2 | 5215,3 | 27787,5 | 21568,9 | 64023,4 | 0,88739742 |
| Netherlands | 738146 | 76580 | 352818 | 237672 | 831809,9 | 0,887397469 |
| Slovenia | 42999,7 | 5633,3 | 21202,9 | 12295,3 | 48466,6 | 0,887202733 |
| U. Kingdom | 2071667 | 224764 | 1009153 | 672940 | 2662484 | 0,778095568 |

*Source:* UNdata. (2021), FRED Economic Data (2021), own evaluation

Table 2

Indicators of international trade and factors of production
at current prices, million units of national currency / current U.S. Dollars

| **Country** | **Products Imports by World from Country** | **Products Exports by Country to World** | **Gross fixed assets** | **Dwellings** | **Human Capital Index** | **Total labour force** | **Relative world demand price** |
|---|---|---|---|---|---|---|---|
| | *USD* | *USD* | *NC* | *NC* | | | $IWC/ECW$ |
| | $IWC$ | $ECW$ | $GFA^{NC}$ | $D^{NC}$ | $HCI$ | $TLF$ | $P$ |
| Austria | 196120 | 159971 | 2349042 | 815690 | 0,793 | 4566592 | 1,2259722 |
| Belgium | 325434 | 430092 | 2263488 | 985852 | 0,757 | 5037594 | 0,7566614 |
| Czechia | 163587 | 182231 | 34115370 | 8920849 | 0,782 | 5419903 | 0,8976903 |
| Denmark | 90240 | 101434 | 12199556 | 5362855 | 0,774 | 2960274 | 0,8896425 |
| Finland | 73898 | 67281 | 1285496 | 541132 | 0,814 | 2709253 | 1,0983487 |
| France | 560699 | 523385 | 11546307 | 5745076 | 0,765 | 30287563 | 1,0712936 |
| Germany | 1350349 | 1446642 | 18533738 | 8741483 | 0,795 | 43288289 | 0,9334369 |
| Israel | 61486 | 61150 | 4211197 | 1968945 | 0,763 | 4025082 | 1,0054947 |
| Italy | 515364 | 507430 | 10264964 | 4667250 | 0,769 | 25985393 | 1,0156357 |
| Luxemburg | 16614 | 13959 | 195028 | 51937 | 0,692 | 293663 | 1,1901999 |
| Netherlands | 462150 | 527908 | 3286019 | 1270176 | 0,8 | 9110882 | 0,8754366 |
| Slovenia | 31781 | 31894 | 244317 | 91419 | 0,788 | 1029046 | 0,996457 |
| U. Kingdom | 413765 | 441847 | 8035643 | 3327072 | 0,781 | 34011299 | 0,9364441 |

*Source:* WITS (2021), OECD.Stat. (2021), World Bank Open Data (2021), own evaluation

Table 3

Efficiency indicators

| **Country** | **Labour intensity** | **Capital intensity** | **Real wage rate** | **Real rental rate** | **Capital-labour ratio** | **The relative price of capital** |
|---|---|---|---|---|---|---|
| | $l$ | $k$ | $w$ | $r$ | $k/l$ | $r/w$ |
| Austria | 10,6668535 | 4,140884126 | 0,055544864 | 0,09841161 | 0,388201087 | 1,771749963 |
| Belgium | 5,75346975 | 2,862312875 | 0,102803504 | 0,142724841 | 0,497493338 | 1,388326617 |
| Czechia | 17,4027238 | 4,991715517 | 0,029609777 | 0,097102736 | 0,286835301 | 3,279414674 |
| Denmark | 6,13750612 | 3,143157621 | 0,102090163 | 0,118804415 | 0,51212293 | 1,163720497 |
| Finland | 9,49830112 | 3,296037833 | 0,059076157 | 0,133152863 | 0,347013407 | 2,253918842 |
| France | 9,58839 | 2,527700111 | 0,067092047 | 0,14111456 | 0,263620912 | 2,103297883 |
| Germany | 8,74463562 | 3,017653367 | 0,072429129 | 0,12149628 | 0,345086233 | 1,677450518 |
| Israel | 8,75614849 | 1,763393224 | 0,059374863 | 0,272261953 | 0,201389141 | 4,58547506 |
| Italy | 10,3708351 | 3,223388683 | 0,051331929 | 0,145078696 | 0,310812839 | 2,82628567 |
| Luxemburg | 3,7777785 | 2,518578102 | 0,149028562 | 0,173511833 | 0,666682311 | 1,164285768 |
| Netherlands | 7,67098331 | 2,73095431 | 0,077890977 | 0,147384238 | 0,356010983 | 1,892186287 |
| Slovenia | 16,67159 | 3,55579225 | 0,037966169 | 0,103224142 | 0,213284531 | 2,718845355 |
| U. Kingdom | 9,34262866 | 2,272841629 | 0,064215213 | 0,176018034 | 0,24327646 | 2,741064406 |

*Source:* own evaluation

### 4.2. International frontiers of efficiency and national distribution functions

As the analysis of the obtained efficiency indicators showed, both technological frontiers, according to the state envelopment method and according to the functions envelopment method, are formed by two countries – Luxembourg and Israel. Both economic frontiers are formed by only the Czech Republic. Their national distribution functions of the net output and average value-added have the form respectively:

$$Q_{LUX} = 0{,}149028562 \cdot L + 0{,}173511833 \cdot K \quad (77)$$

$$V_{LUX} = 3{,}7777785 \cdot W + 2{,}518578102 \cdot R \quad (78)$$

$$Q_{ISR} = 0{,}059374863 \cdot L + 0{,}272261953 \cdot K \quad (79)$$

$$V_{ISR} = 8{,}75614849 \cdot W + 1{,}763393224 \cdot R \quad (80)$$

$$Q_{CZ} = 0{,}029609777 \cdot L + 0{,}097102736 \cdot K \quad (81)$$

$$V_{CZ} = 17{,}40272378 \cdot W + 4{,}991715517 \cdot R \quad (82)$$

The relationship between international efficiency frontiers and the national functions of the leading countries is shown in Figures 4 and 5. These geometric interpretations show a significant difference in the states of the two technology leaders. According to the criterion of technological efficiency, Luxembourg is located in the best parts of its distribution functions – both functions of distributing net output on the technological plane and the function of distributing average value-added on the economic plane. On the contrary, Israel is at the worst area of its national functions.

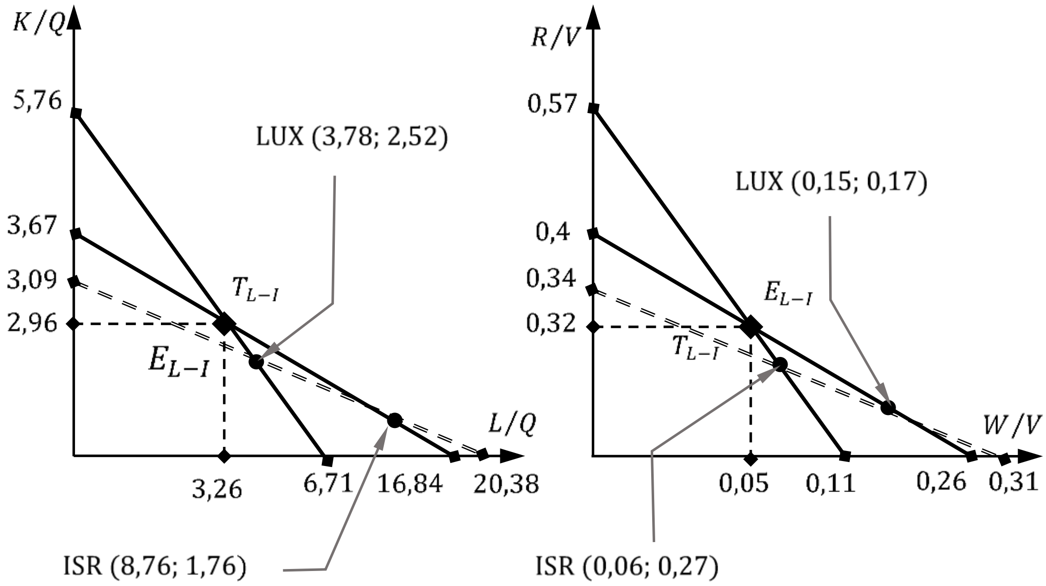


**Figure 4. Technological frontiers of the studied countries in two coordinate systems**

*Source:* own evaluation

On the left side of Figure 4 LUX and ISR, there are points of real technological states of Luxembourg and Israel, respectively, located on the national lines of distribution of the net output. The technological frontier according to the function envelopment method consists of the intersection point of these straight lines $T_{L-I}$ and their segments that are closer to the origin. The technological frontier using the state envelopment method contains a segment of a double hatched straight line $E_{L-I}$ between the points of two countries.

On the right side of Figure 4 LUX and ISR, there are points of real economic states located on national lines of distributions of the average value-added. The technological frontier according to the state envelopment method consists of the intersection point of these straight lines $E_{L-I}$ and their segments that are farther from the origin. The technological frontier using the function envelopment method contains a segment of a double hatched straight line $T_{L-I}$ between the points of two countries.

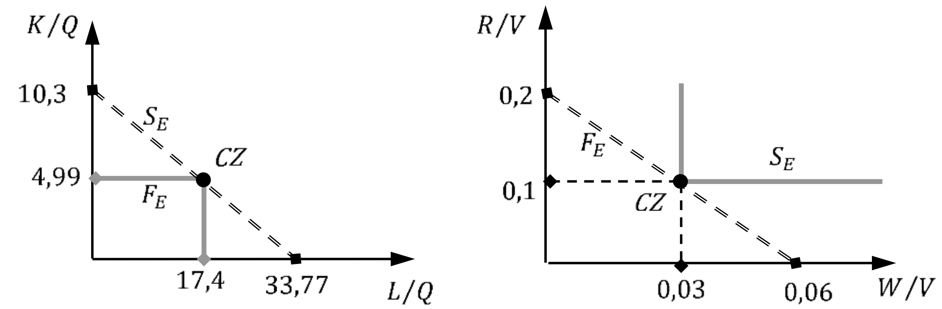


**Figure 5. Economic frontiers of the studied countries in two coordinate systems**

*Source:* own evaluation

On the right side of Figure 5 $CZ$, there is a point of the real economic state of the Czech Republic, located on its national lines of distribution of the average value-added. This straight line represents the economic frontier of the function envelopment method $F_E$. The economic frontier according to the state envelopment method $S_E$ consists of point $CZ$ and two mutually perpendicular semi-straight lines coming out of it.

On the left side of Figure 5 $CZ$, there is a point of the real technological state of the Czech Republic, located on its national lines of distribution of the net output. This straight line represents the economic frontier of the state envelopment method $S_E$. Economic frontier according to the function envelopment method $F_E$ consists of a point $CZ$ and two mutually perpendicular semi-straight lines coming out of it.

### 4.3. Location of the studied countries on the technological and economic planes

Based on the calculated efficiency indicators, it is possible to determine the distances of countries to all frontiers and the distances between opposite frontiers on the same plane in the direction of each country. These values are shown in Table 4.

Table 4

Distances to international efficiency frontiers

| Country | Distance to the technological frontier by using the state envelopment method | Distance to the economic frontier using the function envelopment method | Distance between frontiers on the technological plane | Distance to the economic frontier by using the state envelopment method | Distance to the technological frontier by using the function envelopment method | Distance between frontiers on the economic plane |
|---|---|---|---|---|---|---|
| | $d(\Sigma_{XT})$ | $d(\Phi_{XE})$ | $d(\Sigma_T\Phi_E)_X$ | $d(\Sigma_{XE})$ | $d(\Phi_{XT})$ | $d(\Sigma_E\Phi_T)_X$ |
| Austria | 0,536838993 | 0,829551306 | 0,445335488 | 0,986700003 | 0,472640042 | 0,46635393 |
| Belgium | 0,827732059 | 0,57341266 | 0,474632042 | 0,680349234 | 0,758035122 | 0,515728615 |
| Czechia | 0,405111026 | 1 | 0,405111026 | 1 | 0,384162627 | 0,384162627 |
| Denmark | 0,758838534 | 0,62967483 | 0,477821525 | 0,817332726 | 0,685037563 | 0,559763931 |
| Finland | 0,652676626 | 0,660301618 | 0,430963433 | 0,729257592 | 0,587047172 | 0,428108607 |
| France | 0,776366411 | 0,550970648 | 0,427755105 | 0,688112808 | 0,636774232 | 0,438172505 |
| Germany | 0,711678409 | 0,604532321 | 0,430232601 | 0,799223945 | 0,596083467 | 0,47640418 |
| Israel | 1 | 0,503148162 | 0,503148162 | 0,498692127 | 1 | 0,498692128 |
| Italy | 0,644552031 | 0,645747674 | 0,416217975 | 0,669310786 | 0,597104188 | 0,399648273 |
| Luxemburg | 1 | 0,50455161 | 0,50455161 | 0,55963178 | 1 | 0,559631781 |
| Netherlands | 0,793829687 | 0,547097346 | 0,434302115 | 0,658840713 | 0,690568661 | 0,454974749 |
| Slovenia | 0,508095944 | 0,957987392 | 0,486749508 | 0,94069793 | 0,429550301 | 0,404077079 |
| U. Kingdom | 0,837830743 | 0,536848644 | 0,449788298 | 0,551663567 | 0,730758263 | 0,40313271 |

*Source*: own evaluation

As it follows from the above indicators, on the technological plane, all "outsider" countries can be divided into three groups according to the difference in distances to technological and economic frontiers. Belgium (+0,25), the Netherlands (+0,25), France (+0,23), Denmark (+0,13), Germany (+0,11) and, especially, the United Kingdom (+0,3) are closer to the technological frontier than to the economic one. Austria (–0,29), and even more Slovenia (–0,45), on the contrary, are significantly closer to the economic frontier than to the technological one. Finland and Italy, being almost equidistant from both frontiers, occupy an intermediate position between these two groups. As for the distance between the frontiers on the technological plane, it is the maximum in the direction of Luxembourg and Israel (≈ 0,5) and the minimum in the direction of the Czech Republic (≈ 0,41).

On the economic plane, the distribution of «outsider» countries by distance has a different form. The group of countries that are approximately equally far from both frontiers includes France (–0,05) and the Netherlands (+0,03). Austria (–0,51) and Slovenia (–0,51) are significantly closer to the economic than technological frontier; Denmark (–0,13), Germany (–0,2), Finland (–0,14) and Italy (–0,07) are moderately closer. Belgium (+0,08) is moderately closer to the technological frontier than to the economic one, and the United Kingdom (+0,18) is significantly closer to it. The distances between frontiers on the economic plane have their particularities. This distance is maximum for Luxembourg and Denmark (≈ 0,56) and minimum for the Czech Republic (≈ 0,38).

Luxembourg is the undisputed leader in terms of capital-labour ratio, while in Israel and Slovenia it is three times less. As for the relative price of capital, the situation is diametrically opposite. In Denmark and Luxembourg, this price is almost four times lower than in Israel.

A geometric interpretation of the location of countries on the technological and economic plane is shown in Figures 6 and 7.

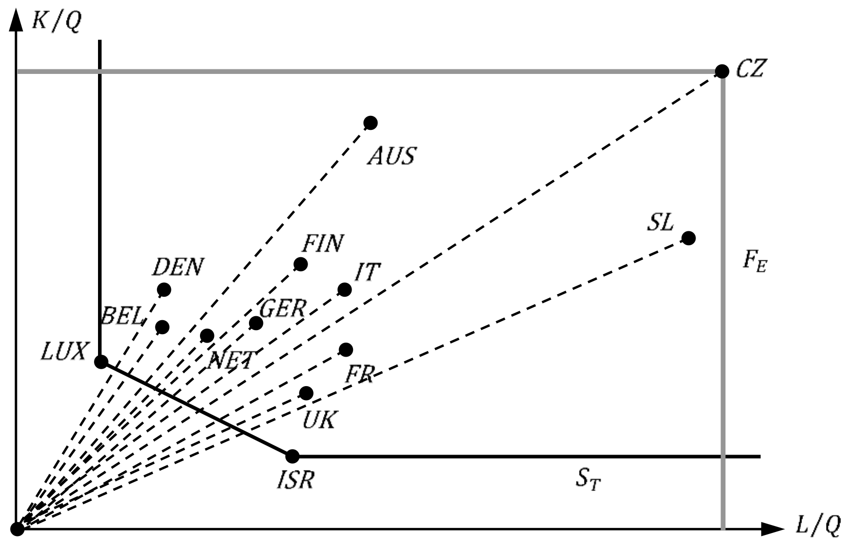


**Figure 6. Location of the studied countries on the technological plane**

*Source:* own evaluation

In Figure 6 $S_T$ is a technological frontier according to the state envelopment method, $F_E$ is an economic frontier according to the method of distribution functions envelopment, the average value-added. Dashed lines show the rays of the country's capital-labour ratio.

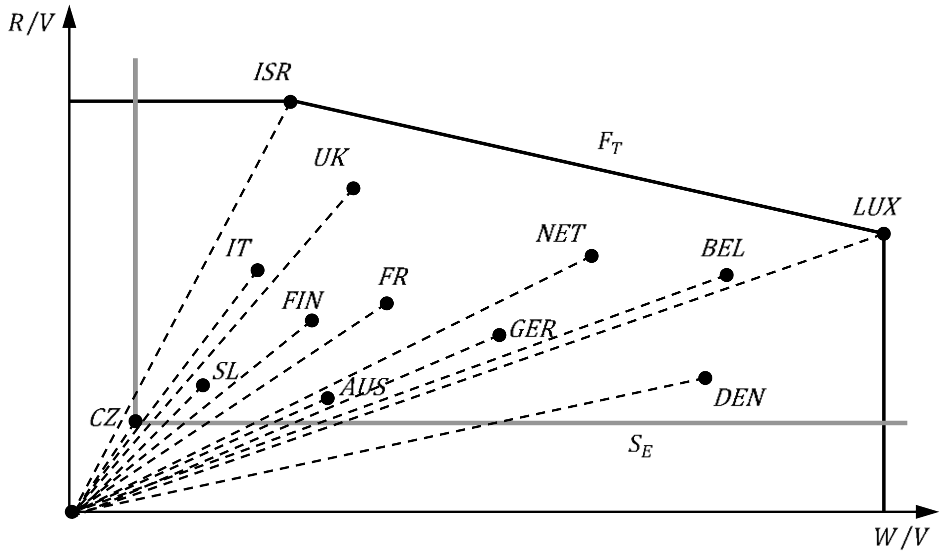


**Figure 7. Location of the studied countries on the economic plane**

*Source*: own evaluation

In Figure 7 $S_E$ is an economic frontier by the state envelopment method, $F_T$ is a technological frontier by the method of distribution functions envelopment on the net output. Dashed lines show the rays of the relative capital price.

### 4.4. Analysis from the point of view of market theory

Based on the theory markets factor, this arrangement of countries on both planes can be explained as follows. There are correlations between the real prices of factors of production and the intensity of their use, which are described by the following regression equations:

$$\hat{w} = 0{,}6114562 \cdot l\hat{}(-1{,}02117996) \qquad R^2 = 0{,}9754 \tag{83}$$

$$\hat{r} = 0{,}40545532 \cdot k\hat{}(-0{,}98411594) \qquad R^2 = 0{,}8697 \tag{84}$$

These equations are logically interpreted as functions of demand for factors of production, $\hat{w}$ and $\hat{r}$ as demand prices for labour and capital, respectively. The three countries forming the frontiers of efficiency are located along with the graphs of these functions, and on the labour market plane this descending series looks like: Luxembourg – Israel – Czech Republic, and on the capital market plane: Israel – Luxembourg – Czech Republic.

On the other hand, between the capital-labour ratio $\kappa = k/l$ and the relative price of capital $\rho = r/w$ there is also a correlation:

$$\hat{\kappa} = 0{,}59752283 \cdot \rho\hat{}(-0{,}77447313) \qquad R^2 = 0{,}8207 \tag{85}$$

This property can be explained by the fact that relatively more expensive factor of production and is used relatively less. Consequently, the angle between the ray of the country's capital-labour ratio and the axis $0l$ is very close in value to the angle between the ray of the relative price of its capital and axis $0r$. Thus, for Israel and the Czech Republic, the capital-labour ratio is almost equal to the relative price of labour $\omega = w/r$, and for Luxembourg, it is approximately equal to $77\%$:

$$\kappa_{ISR} = 0{,}2014 \qquad \omega_{ISR} = 1/\rho_{ISR} = 1/4{,}5855 = 0{,}2181 \tag{86}$$

$$\kappa_{CZ} = 0{,}2868 \qquad \omega_{CZ} = 1/\rho_{CZ} = 1/3{,}2794 = 0{,}3049 \tag{87}$$

$$\kappa_{LUX} = 0{,}6667 \qquad \omega_{LUX} = 1/\rho_{LUX} = 1/1{,}1643 = 0{,}8589 \tag{88}$$

A geometric interpretation of the relation between the observed efficiency frontiers and the theoretical functions of demand for factors of production is shown in Figure 8.

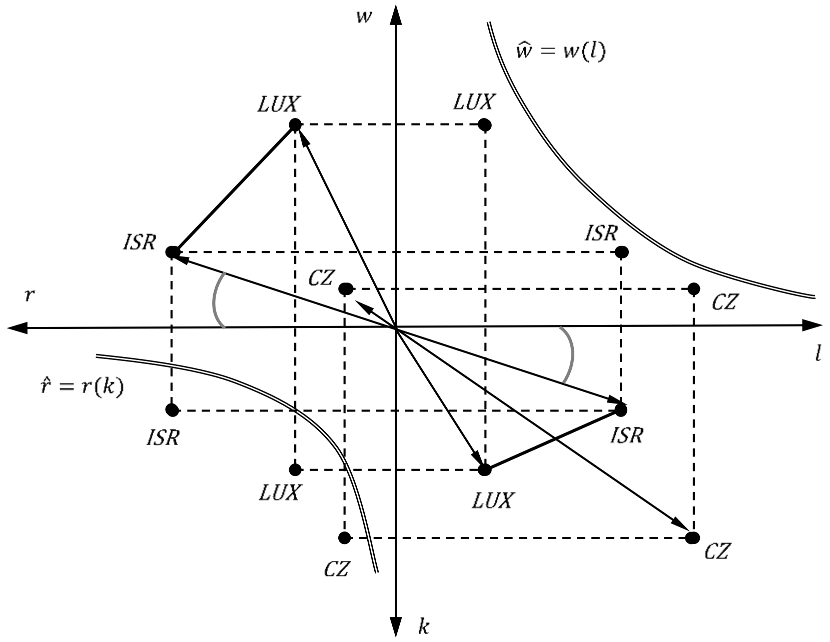


**Figure 8. The linkage between efficiency frontiers and demand functions for factors of production**
*Source:* own evaluation

In Figure 8, in the lower right quadrant, the arrow lines show the rays of the capital-labour ratio availability, and in the upper-left quadrant, there are the rays of the relative price of capital.

## 5. CONCLUSION

The results of the analysis grounded that efficiency indicators for both technological frontiers (due to the method of countries' states envelopment and method of national distribution functions envelopment) are formed by two countries – Luxembourg and Israel. However, only the Czech Republic formed both economic frontiers. Luxembourg has the best position with its distribution functions, however, Israel has the worst. Luxembourg, the United Kingdom and Belgium have the best combinations of production intensity factors among the studied European countries and the worst combinations of real factor prices, while the other three countries – the Czech Republic, Slovenia and Austria, on the contrary, have the best combinations of real factor prices and the worst combinations of production factor intensities. Thus, the geographical axis on the European continent is traced in the direction of «northwest-southeast».

The analysis based on the market theory determined that the technological frontiers of the investigated group of countries are envelopments of point projections that form the demand curve on the planes of the labour market and the capital market. So, at least for this group of countries, only firms that use the production factors are main actors, that define macroeconomic entities (respectively, the factors' owners aren't the main actors).

The authors proposed to use this model for prompt comparative analysis of selected national economies effectiveness based on available official statistics data.

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