Market anomalies and data persistence: 
The case of the day-of-the-week effect

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Abstract. This paper investigates the degree of persistence in the financial markets’ data during different days of the week over the last twenty years. This allows taking a brand new look on the day-of-the-week effect and providing additional evidence against the efficient market hypothesis. The variety of the financial markets includes developed and emerging stock markets, FOREX, commodity and cryptocurrency markets. To measure the level of persistence the R/S analysis is used. The findings indicate that the level of persistence is different for different days of the week. This is inconsistent with the Efficient Market Hypothesis: data do not follow a random walk; and there can be indirect evidence in favor of the day-of-the-week effect. Conclusions on non-randomness of the data are important, because they allow choosing the best model to describe price dynamics so that to increase the predictive power of the existing models. Differences in the long-memory properties of the market data during different days of the week is an important finding that can lead to better understanding of the behavior of financial markets. High level of persistence implies data predictability, and therefore suggests that trend following technics can be applied to make profits from trading.

Keywords: market efficiency, anomaly, long memory, persistence, day of the week effect, R/S analysis.

JEL Classification: C22, G12
1. INTRODUCTION

According to the Efficient Market Hypothesis (EMH), prices at financial markets should follow a random walk process and there should be no systemic patterns in the price behavior (Fama, 1965; Samuelson, 1965). Nevertheless, during the last 30-40 years, there has been found a lot of inconsistencies between the real life and the EMH theory. They include market anomalies (Urquhart & McGroarty, 2014; Seif et al., 2017, Caporale & Plastun, 2017), fat tails in price distribution (Mandelbrot, 1963), price bubbles and anti-bubbles (Johansen & Sornette, 1999), market overreactions and underreactions (De Bondt & Thaler, 1985), long memory effect (Greene & Fielitz, 1977; Helms et al., 1984, Jacobsen, 1995) etc. Tokić et al. (2018), Kumaraswamy et al. (2019) show that nowadays the efficient market hypothesis can’t hold even in developing countries. The most recent study by Fayez and Amani (2019) shows that there is a positive relationship between price returns and trading volume in Kuwait Boursa, and this conclusion is inconsistent with the EMH.

These deviations from theory can be explained by the presence of information asymmetry and its consequences (moral hazard, adverse selection etc.), transaction costs at financial markets, different sorts of irrational behavior among investors like panic, fear, greed, herd instinct, etc. (Plastun et al., 2019a). Another possible reasons for this is the evolution of financial markets: they move from less efficient to more efficient states (Njuguna, 2016).

Despite a very significant number of academic papers devoted to long memory analysis, as well as market anomalies, none of them put these two aspects together (the only exception is the paper by Plastun et al., 2019b).

The present paper aims to find whether the degree of persistence in data sets (normal and anomaly related) differs. This might be indirect evidence in favor of abnormal price behavior related to market anomalies and also will be evidence against the market efficiency theory.

To do this, data persistence is analyzed for different financial assets and different financial markets. The most liquid financial markets nowadays are analyzed: stock market (both developed and emerging ones), FOREX, commodity and cryptocurrency markets. Data samples cover the period from 1999 till the end of 2018. We will explore the day of the week anomaly – this is one of the most famous anomalies. The main hypothesis tested in this paper is as follows: “The level of persistence in financial data is different during different days of the week”. To test this hypothesis, the R/S analysis with the Hurst exponent is used.

The results of this analysis can be interesting to academia, because they expand the existing knowledge about the nature of financial markets. Also, they can be used by practitioners, because these results allow detecting market inefficiencies and exploiting them in practice.

The layout of the paper is as follows. Section 2 provides a brief review of literature on market anomalies and market persistence; Section 3 describes data and methodology; Section 4 contains the empirical results; Section 5 provides some concluding remarks.

2. LITERATURE REVIEW

Since 80-s of the XX\textsuperscript{th} century, hundreds and thousands of papers devoted to inconsistencies between the real life and the EMH were published. Most of them were related to so-called “market anomalies” (situations when certain patterns or typical behavior is observed in price dynamics). According to Jensen (1978), anomalies are real only when they generate excess returns.

Currently there are dozens of detected anomalies. Some of them are associated with prices and returns (possibility of profitable trading based on contrarian or momentum strategies, size effect and value investing, etc.), other with trading volume and volatility (panic effect in the markets, existence of price bubbles, etc.). There is another class of market anomalies, so called “calendar anomalies” related to seasonal patterns. They
include day-of-the-week effect, January effect, month-of-the-year effect, Halloween effect, turn-of-the-month effect, Holiday effect, turn-of-the-year effect, etc.

Calendar anomalies are widely discussed in the academic literature. They are explored in the different financial markets (stock, FOREX, commodities, cryptocurrency market, etc.), different countries (developed and emerging), different time horizons (most of the papers are concentrated on the most recent data, but there are some of them which cover the whole available historical data).

Existing evidences are mixed. Possible explanation for the differences in results is provided by Plastun et al. (2019). They conduct a comprehensive investigation of calendar anomalies (day of the week effect, turn of the month effect, turn of the year effect, and the holiday effect) in the US stock market for the period from 1900 till 2018 and show that anomalies evolve in time. The ‘golden age’ of calendar anomalies was in the middle of the 20th century and currently they have almost disappeared from developed markets.

Shakila et al. (2017) show that calendar anomalies are disappearing not only from developed markets, but also from emerging. Using semi-monthly effect as an example of calendar anomaly and Bombay Stock Exchange as an object of analysis they cannot provide any evidence for the presence of semi-monthly effect. Similar conclusions are provided by Ebrahim et al. (2016), but for the case of Tehran Stock Exchange.

One of the most frequently observed calendar anomalies is the day of the week effect and its forms: Monday Effect, Friday Effect. Cross (1973) found that prices in the US stock market tend to increase on Fridays and decrease on Mondays. Further investigation by French (1980) showed that returns on Mondays usually are negative. After this influencing paper, hundreds of similar papers appeared searching for the day-of-the-week effect in different financial markets and assets: 19 developed stock markets all over the world (Agrawal & Tandon, 1994), emerging stock markets (Caporale et al., 2016), commodity markets (Singal and Tayal, 2014), FOREX (Dao et al., 2016) and cryptocurrency market (Caporale & Plastun, 2018) etc.

Plastun et al. (2019) show that the “golden age” of day-of-the-week effect was in the 70-80s of the XXth century and currently it has almost disappeared. Overall current evidences in favor of the day-of-the-week effect in the financial markets are mixed. This can be caused by the differences in analyzed data sets, data periods, types of the financial markets, etc. One more possible explanation for these differences is the use of different methodology.

To detect the day-of-the-week effect, a variety of statistical methods and econometrical models are usually used: t-tests, ANOVA analysis, Kruskall-Wallis and Mann-Whitney tests, regression analysis with dummy variables, jump diffusion models, correlation analysis, Chow breakpoint and Bai-Perron Tests, spectral analysis.

Despite wide range of methodological tools used to detect the day-of-the-week effect and a lot of results describing almost all possible markets and assets, still there was no attempt to analyze long memory properties of the financial data during different days of the week. In theory, this can shed a light on reasons for the day-of-the-week effect existence or at least can act as additional methodological tool to detect market anomalies (not only the day-of-the-week effect, but a lot of others as well).

Information about market persistence (long memory) is very important to understand the nature of the processes in the financial markets. For example, the presence of long memory evidences about non-random nature of the data series. This in turn is against the Random Walk Hypothesis and the EMH. The level of persistence helps to identify the best models for the data analysis. High persistence in data shows that auto regression and trend models can be used to predict future data. Long memory analysis is widely used in the financial markets analysis. Greene and Fieltz (1977), Booth, Kaen, and Koveos (1982), Helms et al. (1984), Onali and Goddard (2011), Niere (2013), Caporale and Gil-Alana (2013), Skrodzka (2015) Blajer-Golębiewska and Kos (2016), and others provide evidence of persistence and long memory in the stock markets (both developed and emerging), FOREX, commodities and cryptocurrency markets. Ali et
al. (2017) test a relationship between long memory features in returns and volatility of Dhaka Stock Exchange market and find long memory properties in Bangladesh stock returns.

Caporale et al. (2018) examine persistence in the cryptocurrency market and find evidences in favor of long memory in returns (a positive correlation between past and future values). So long memory properties of the data are widely explored in different financial markets.

Nevertheless, long memory and persistence analysis were not used for the anomalies detection purposes. The only exception is attempt by Plastun et al. (2019b) who based on US stock market data explored persistence for the different days of the week. Using the R/S analysis for the US stock market data during 1900–1969 they show that in general, the highest values of the Hurst exponent were observed on Mondays (evidence in favor of persistence in data). But since 1970, the level of persistence decreased and was close to the values typical to random data.

The aim of this paper is to fill the gap in the existing literature on market anomalies and persistence analysis. Results of this research are important, because they allow to understand deeper and better the nature of the financial markets; they can be useful for the practitioners, as well as because the day of the week effect can affect investors in deciding portfolio selection, profit management, and overall investment strategy; and finally, using the most recent data we can find the newest evidences about the day-of-the-week effect.

3. DATA AND METHODOLOGY

This paper explores the degree of persistence during different days of the week. To do this, daily data from various financial markets is used. FOREX is presented by two currency pairs namely EURUSD and USDJPY over the period from January 1, 1999 till December 31, 2018. Stock market instruments include main indexes from the US stock market: Dow Jones Industrial, SP500 and NASDAQ (in all these cases the sample goes from January 1, 2004 till December 31, 2018). Also to see whether there are differences for the case of emerging markets additional data from Russian (MICEX index data from January 1, 2004 till December 31, 2018) and Ukrainian (PFTS index data from January 1, 2005 till December 31, 2018) stock markets. Commodity market is presented by Gold and Oil daily data. Both samples go from January 1, 2004 till December 31, 2018. As an additional object of analysis a cryptocurrency market data is incorporated. Data from this market include daily returns for the following cryptocurrencies: BitCon, Ripple, Dash and LiteCoin. All crypto data samples start from 01.01.2014 to 31.12.2018.The data sources are Yahoo! Finance (https://finance.yahoo.com) for the cases of FOREX, Commodity and US stock markets; PFTS Stock Exchange (http://www.pfts.ua/) for the case of Ukrainian stock market; Moscow Exchange (https://www.moex.com/en/) for the case of Russian stock market; CoinMarketCap (https://coinmarketcap.com/coins/) for the case of cryptocurrency market.

Hypothesis to be tested in this paper is as follows: “The level of persistence in the financial data is different during different days of the week”.

To measure persistence, we apply R/S analysis with the following algorithm (Hurst, 1951):

1. An original dataset with the length $L$ is converted into dataset with the length $T = L - 1$ using logs and converting prices into returns:

$$T_i = \log \log \left(\frac{P_i}{P_{i+1}}\right), i \in [1..(L - 1)]$$

2. Period $T$ is divided into $N$ sub-periods with length $n$. After that each sub-period is identified as $T_n$ ($n \in [1, 2, 3, \ldots, N]$) and the average value is calculated as follows:

$$\text{Aver}_n = \frac{1}{T} \sum_{n=1}^{T} T_n$$
3. Then accumulated deviations $AD_{k,a}$ for each sub-period are calculated as:

$$AD_n = \sum_{i=1}^{n}(T_i - \text{Aver}_i)$$  (3)

After this the range is defined for each sub-period ($T_n$):

$$R_T = \text{maximim}(AD_n) - \text{maximim}(AD_n)$$  (4)

4. Next for each sub-period $T_n$ the standard deviation ($\sigma_{T_n}$) is calculated:

$$\sigma_{T_n} = \sqrt{\frac{1}{T} \sum_{i=1}^{n}(T_i - AD_i)^2}$$  (5)

5. The average R/S for length $n$ is defined as:

$$R/S_n = \frac{1}{N} \sum_{i=1}^{N} \frac{R_T}{\sigma_{T_n}}$$  (6)

6. The length $T$ is transformed into $(L - 1)/T$, which should be an integer, and Stages 1 - 6 are repeated until $T = (L - 1)/2$.

7. The angle of the regression line $\log(R/S) = \log (c) + H \log (t)$ is an estimate of the Hurst exponent $H$ (Hurst, 1951).

The Hurst exponent ($H$) lies in the interval $[0, 1]$. There are three types of data sets:

- anti-persistent ($H < 0.5$) – returns have negative correlations;
- non-persistent ($H = 0.5$) – no long memory detected, returns are random, they are uncorrelated;
- persistent ($H > 0.5$) – long memory is present in data; returns are highly correlated.

4. EMPIRICAL RESULTS

The results of the R/S analysis for the stock market are presented in Table 1 (US stock market) and Table 2 (emerging markets).

### Table 1

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dow Jones Industrial Index</td>
<td>0.50</td>
<td>0.62</td>
<td>0.49</td>
<td>0.54</td>
<td>0.58</td>
<td>0.55</td>
</tr>
<tr>
<td>SP500 Index</td>
<td>0.52</td>
<td>0.54</td>
<td>0.61</td>
<td>0.50</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>NASDAQ Index</td>
<td>0.58</td>
<td>0.56</td>
<td>0.64</td>
<td>0.49</td>
<td>0.60</td>
<td>0.58</td>
</tr>
</tbody>
</table>

As can be seen, market persistence differs for the different days of the week. For the cases of SP500 and NASDAQ Indices, data are much higher on Wednesdays. Also Friday is the day with higher persistence (this observation is true for the Dow Jones Industrial Index and NASDAQ Index). So these two days are the most appropriate for the anomalies existence. Also Tuesday is abnormal day for the case of Dow Jones Industrial Index.

To see whether or not there are differences in persistence behavior for the case of emerging markets, Russian and Ukrainian stock markets are analyzed. Results are presented in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>PFTS Index</td>
<td>0.54</td>
<td>0.73</td>
<td>0.60</td>
<td>0.65</td>
<td>0.61</td>
<td>0.62</td>
</tr>
<tr>
<td>MICEX Index</td>
<td>0.60</td>
<td>0.64</td>
<td>0.57</td>
<td>0.41</td>
<td>0.56</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Results for the Ukrainian stock markets clearly show that it is less efficient: average Hurst exponent is 10-20% higher than in the US stock market. Average value of $H=0.62$ means that PFTS Index data do not follow a random walk, which is against the Efficient Market Hypothesis.

The most persistent day for the PFTS and MICEX Indices is Tuesday (this is in charge with the Dow Jones Industrial Index estimates).

The results for the FOREX are presented in Table 3.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>EURUSD</td>
<td>0.52</td>
<td>0.50</td>
<td>0.58</td>
<td>0.55</td>
<td>0.62</td>
<td>0.55</td>
</tr>
<tr>
<td>USDJPY</td>
<td>0.50</td>
<td>0.59</td>
<td>0.59</td>
<td>0.55</td>
<td>0.57</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Persistence in FOREX differs depending on the day of the week. The highest level of persistence is observed on Fridays and Wednesdays. Returns on Mondays are the lowest and close to the random data results.

Estimates of the Hurst exponent for the commodity market are presented in Table 4.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.54</td>
<td>0.58</td>
<td>0.52</td>
<td>0.51</td>
<td>0.54</td>
<td>0.54</td>
</tr>
<tr>
<td>Oil</td>
<td>0.60</td>
<td>0.57</td>
<td>0.61</td>
<td>0.58</td>
<td>0.53</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Results for the Gold and Oil are not synchronized (the only exceptions are Friday and Tuesday). Mondays and Wednesdays are the days with the highest persistence for the case of Oil and Tuesdays have abnormally high Hurst Exponent estimations for the case of Gold.

The results of the R/S analysis for the cryptocurrency market are presented in Table 5.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bitcoin</td>
<td>0.65</td>
<td>0.66</td>
<td>0.48</td>
<td>0.63</td>
<td>0.53</td>
<td>0.59</td>
</tr>
<tr>
<td>Litecoin</td>
<td>0.60</td>
<td>0.55</td>
<td>0.68</td>
<td>0.71</td>
<td>0.61</td>
<td>0.63</td>
</tr>
<tr>
<td>Ripple</td>
<td>0.48</td>
<td>0.67</td>
<td>0.57</td>
<td>0.71</td>
<td>0.65</td>
<td>0.62</td>
</tr>
<tr>
<td>Dash</td>
<td>0.54</td>
<td>0.58</td>
<td>0.64</td>
<td>0.55</td>
<td>0.68</td>
<td>0.60</td>
</tr>
</tbody>
</table>

As can be seen cryptocurrency market in general is rather inefficient. Level of persistence is unstable during a week, but there are no stable patterns detected. For example, in case of Bitcoin, days with high persistence are Mondays and Tuesdays. For the case of Litecoin, these are Wednesdays and Thursdays. Tuesdays and Thursdays are the days with the highest persistence for the case of Ripple. As for the dash, these days are Wednesdays and Fridays.

In order to summarize the results, we choose two days from each data set with the highest persistence. The results are presented in Figure 1.

Figure 1 defines the most favorable days for the presence of long memory properties in data. These days are Tuesday and Wednesday. It doesn’t necessarily mean the presence of the calendar day-of-the-week anomaly (returns on these days significantly higher or lower comparing with the other days of the week), still it shows thing, which shouldn’t be existed according to the EMH. In random data, there should be no
differences in persistence during different days of the weeks and certainly there should be no favorable days for the increase of the persistence.

Figure 1. Summary of results

Overall results show that persistence is not stable during a week. This can be indirect evidence in favor of market anomalies and possible price patterns. This in turn is against the market efficiency. Nevertheless, we can’t find any stable pattern in persistence behavior for a specific day of the week. Looks like this is very individual case and depends on type of the market, as well as the asset type. Still persistence on Tuesdays and Wednesdays is higher than its average values. So we might conclude that these two days are the most favorable for the data prediction.

The most important implications from these results are:
1. The difference of persistence in the data for a specific day of the week can be indirect evidence in favor of the day-of-the-week market anomaly.
2. High values of the Hurst exponent evidence in favor of the long-memory in the analyzed data overall and specific days of the week is in charge with the Fractal Market Hypothesis. It means that price data in the most of the analyzed financial markets can be informative about future and forecasting models based on past data can be effective. It also means that classic asset pricing models (CAPM, Black–Scholes, Markowitz, etc.) can generate a lot of mistakes and inconsistencies with the real life.
3. The presence of persistence in the data gives a rationale for the use of specific trading technics based on the day-of-the-week effect. For example, high persistence evidence is favor of the possibility for the use of trend trading strategies. The instability of the long memory properties of the data for the different days of the week explains the necessity of switching between trading strategies.
4. Non-randomness of the data allows choosing the best model to describe price dynamics. As a result, the predictive power of the models can be increased.

5. CONCLUSIONS

Market anomalies analysis is very interesting object of analysis even nowadays. Despite a lot of empirical evidences from different markets there are still unexplored moments both of methodological and empirical nature. One of them is long memory properties of the normal and anomaly related data sets. This
paper uses R/S analysis to define the level of persistence in various financial markets (developed and emerging stock markets, FOREX, commodity and cryptocurrency markets) for the different days of the week over the last 20 years.

The evidence suggests that the level of persistence is different for the different days of the week. This is against the Efficient Market Hypothesis (data do not follow a random walk) and can be indirect evidence in favor of the day-of-the-week effect. High values of the Hurst exponent evidence in favor of the presence of long-memory properties in the analyzed data overall and specific days of the week is in charge with the Fractal Market Hypothesis. Conclusions on non-randomness of the data are important, because allow choosing the best model to describe price dynamics and to increase the predictive power of the models.

The difference in the long memory properties of market data during different days of the week is important, because it can lead to a better understanding of the nature of the financial markets. High level of persistence implies data predictability, and therefore suggests that trend trading strategies can be used to generate abnormal profits in the financial markets.

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